



Register Number:

Date:

ST. JOSEPH'S COLLEGE (Autonomous), BENGALURU - 27
M.Sc Mathematics-II Semester
Semester Examination: April 2022
(Examination conducted in July 2022)
MT-8318: Theory of Numbers

Time: $2\frac{1}{2}$ Hours

Max. Marks:70

The paper contains one page

ANSWER ANY SEVEN OF THE FOLLOWING QUESTIONS 7*10=70

1. For $n \geq 0$, prove that $\sum_{d|n} \phi(d) = n$ [10]
2. Prove that, for $n \geq 1$, $\phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$ [10]
3. If both g and $f * g$ are multiplicative, then prove that f is also multiplicative. [10]
4. Write the Bell series for Mobius function, Euler totient function, Completely multiplicative function, Unit function and Liouville's function. [10]
5. Given a prime p , let $f(x) = c_0 + c_1x + \dots + c_nx^n$ be polynomial of degree n with integer coefficients such that c_n not congruent to $0 \pmod{p}$. Then show that the polynomial congruence $f(x) \equiv 0 \pmod{p}$ has at most n solutions.
6. Solve for x ,
 $x \equiv 2 \pmod{3}$
 $x \equiv 4 \pmod{5}$
 $x \equiv 6 \pmod{7}$ [10]
7. State and prove Euler's Criterion for quadratic residue. [10]
8. State and prove Gauss lemma. [10]
9. Find all the primitive roots for the prime $p = 11$ [10]
10. Write all the partitions of 9. [10]