Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.SC MATHEMATICS - II SEMESTER
SEMESTER EXAMINATION: APRIL, 2022
(Examination conducted in July 2022)

MT 8418 – PARTIAL DIFFERENTIAL EQUATIONS

Time: 2.5 Hours **Max. Marks:** 70

1. The paper contains **ONE** printed page.
2. Attempt any **SEVEN FULL** questions.
3. Every question carries **TEN** marks.

1. a) Find the general integral of the partial differential equation $y^2p - xyq = x(z - 2y)$.
   b) Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$. (4+6)
2. Solve the Cauchy problem by the method of characteristics $(y + u)u_x + yu_y = x - y$ with $u = 1 + x$ on $y = 1$.
3. Reduce the given PDE $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$ to its canonical form and hence find the general solution.
4. a) Solve $(D^2 + 2DD' + D'^2)z = e^{2x+3y}$.
   b) Solve $DD'(D - 2D' - 3)z = 0$. (5+5)
5. Solve $r + (a + b)s + abt = xy$ using Monge's method.
6. Obtain the general solution three-dimensional wave equation in spherical polar co-ordinates.
7. Solve $\dfrac{\partial u}{\partial t} = c^2\dfrac{\partial^2 u}{\partial x^2}$, $0 < x < L$ and $t > 0$, subjected to the conditions $u(x, 0) = f(x)$, $0 < x < L,\ u(0, t) = u(L, t) = 0, t > 0.$ where c^2 is the thermal conductivity.
8. Find the solution of the non-homogenous wave equation $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 z}{\partial t^2}$ and $z(x, 0) = f(x), z_t(x, 0) = g(x)$ using Reimann-Volterrra method.
9. Solve the Dirichlet problem $\nabla^2 u = 0$, $0 < x < \pi$, $0 < y < \pi$ subjected to the boundary conditions $u(x, 0) = x$, $u(x, 1) = 0$, $u(0, y) = 0$, $u(1, y) = 0$.
10. a) Define Green's function for the boundary value problem.
    b) Find the Green's function of $u'' + k^2u = 0$ with the boundary condition $u(0) = u(1) = 0$, $k \neq n\pi$. (2+8)