Register Number:

Date:

St. Joseph's College (Autonomous), Bangalore-560027
M.Sc Mathematics - II Semester
Semester Examination: April 2022
(Examination conducted in July 2022)
MT 8421: Partial Differential Equations

Time: 2.5 Hours **Max. Marks:** 70

1. This paper contains ONE printed page.
2. Attempt any **SEVEN FULL** questions.

1. a) Form a PDE by eliminating the arbitrary constants a and b from $z = (x-a)^2 + (y-b)^2$ **[3m]**
   b) Find the integral surface of the PDE $2y(z-3)p + (2x-z)q = y(2x-3)$ which pass through the circle $x^2 + y^2 = 2x$ and $z = 0$. **[7m]**
2. Find the canonical form of the PDE $y^2 r - 2xys + x^2 t = \left(\frac{y^2}{x}\right) p + \left(\frac{x^2}{y}\right) q$,
   where $x \neq 0$, $y \neq 0$ and hence find the general solution. **[10m]**
3. a) Classify the PDE $yr + (x+y)s + xt = 0$ into hyperbolic/ parabolic/ elliptic type depending on the conditions given below:
   i) when $x = y$ ii) when $x \neq y$ **[3m]**
   b) Solve $(D - D'^2)z = cos(x - 3y)$. **[7m]**
4. (a) Solve $x^2 r - y^2 t - yq + xp = 0$. **[5m]**
   (b) Solve $xs + q = 4x + 2$. **[5m]**
5. Solve the given PDE $r = a^2 t$ using Monge's method. **[10m]**
6. Obtain the general solution of one dimensional wave equation using the method of separation of variables. **[10m]**
7. Deduce the D'Alembert's solution for the given Cauchy problem: $z_{tt} - c^2 z_{xx} = 0$, where $-\infty < x < \infty$, $t > 0$ subjected to the conditions $z(x,0) = f(x)$ and $z_t(x,0) = g(x)$. **[10m]**
8. Solve the Neumann problem $\nabla^2 u = 0$, where $0 < x < \pi$, $0 < y < \pi$ subjected to the boundary conditions, $u_y(x,0) = cos(x)$, $u_y(x,\pi) = 0$, $u_x(0,y) = 0$, $u_x(\pi,y) = 0$. **[10m]**
9. Derive the solution of three dimensional Laplace equation in cylindrical co-ordinates. **[10m]**
10. Solve using the method of eigen function expansion, $u_{tt} - u_{xx} = \pi^2 sin(\pi x)$,where $0 < x < 1$, $t > 0$, subjected to the boundary conditions $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = \pi$ and $u_t(x,0) = 2\pi sin(2\pi x)$. **[10m]**