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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc. MATHEMATICS– V SEMESTER
SEMESTER EXAMINATION: NOVEMBER 2020
MT5218- MATHEMATICS VI

Time- $2\frac{1}{2}$ Hrs. **Max Marks-70**

This paper contains THREE parts and TWO pages.

I. Answer any FIVE of the following questions. **(5×2=10)**

1. Show that $|z-1|^2+|z+1|^2=4$ represents a unit circle.
2. Evaluate $\lim_{z\to i}\left(\frac{z^3+i}{1-zi}\right)$
3. Evaluate $\int_0^{3+i} z^2\ dz$ along the line $3y=x$.
4. Verify C-R equations for $f(z)=z-\bar{z}$.
5. If $\vec{F}=x^2y\hat{i}+2xz\hat{j}+2yz\hat{k}$, find curl($\vec{F}$) at (1,1,1)
6. Show that the vector field $\vec{F}=2x^2z\hat{i}-10xyz\,\hat{j}+3xz^2\hat{k}$ is Solenoidal.
7. Check whether the function $\phi=x^2-y^2+2xy$ is Harmonic.
8. If $\vec{r}$ represents the position vector of a point P, then show that div($\vec{r}$) $=3$ and curl($\vec{r}$) $=0$.

II. Answer any SEVEN of the following questions. **(7×6=42)**

9. Show that $arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$ represents a circle. Find its centre and radius.
10. Discuss the transformation $w=\sin z$
11. Find the bilinear transformation which maps $1,-i,-1$ in the z-plane onto $0,\ i,\ \infty$ in the w-plane. Also find the invariant points under this bilinear transformation.

12. State and prove sufficient conditions for $f(z) = u + iv$ to be analytic in a domain D.

13. Show that $u = e^x \cos y + xy$ is harmonic and find its harmonic conjugate v.

14. Find the analytic function whose real part is $\left(r + \frac{1}{r}\right)\cos\theta$.

15. State and prove Cauchy's integral formula.

16. Evaluate $\oint_c \frac{dz}{e^z(z-1)^3(z+3)}$ where c is the circle $|z| = 2.1$.

17. a) Evaluate $\oint_c \frac{dz}{(z^2+4)^2}$ where c is the circle $|z + i| = 2$.

b) Evaluate $\oint_c \frac{\sin \pi z}{(z-\pi)}\, dz$ where c is the circle $|z| = 4$. (4+2)

III. Answer any THREE of the following questions. (3×6=18)

18. a) Find the directional derivative of $\phi = xy + yz + xz$ at the point (1, 2, 0) in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$

b) Evaluate $grad\left(\frac{e^{xz}}{\sqrt{x^2+y^2}}\right)$ (4+2)

19. Show that $\vec{F} = (e^x cosy + yz)\hat{i} + (xz - e^x siny)\hat{j} + (xy + z)\hat{k}$ is conservative and also find its scalar potential.

20. Find the equation of the tangent plane and normal to the surface $xyz = 4$ at the point $(1,2,2)$.

21. If $\vec{F}$ is a differentiable vector function and ϕ is a differentiable scalar function, then prove that

i) $div(grad\phi) = \nabla^2\phi$

ii) $curl(\phi F) = \phi curlF + (grad\phi \times F)$ (2+4)

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