ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. Physics: II SEMESTER
END SEMESTER EXAMINATION: APRIL 2022
(Examination conducted in July 2022)
PH8421: Quantum Mechanics-I

Time: $2\frac{1}{2}$ **hours** **Maximum marks: 70**

This question paper has 2 printed pages and 2 parts

PART A

Answer any **FIVE** of the following questions. Each question carries 10 marks. $[5 \times 10 = 50]$

1. Obtain the expression for the Eigen energies and normalized Eigen functions for a particle trapped in an infinite potential well. [10]

2. Using the Associated Legendre function and Rodrigues formula construct the angular wave functions for $l = 0,\ m = 0$, and $l = 2,\ m = 1$. Where l and m are the Azimuthal and magnetic quantum numbers respectively. Check that they are normalized and orthogonal. [10]

3. Obtain Bohr's formula for the allowed energies for the Hydrogen atom from the radial wave equation given below.

$$\frac{\hbar^2}{2m}\frac{d^2U}{dr^2} + \left[V(r) + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]\ U = E\ U$$

[10]

4. (a) Use Rodrigues formula to derive Hermite polynomials H_3 and H_4. [5]
   (b) Show that the eigen values associated with observables are real. What are the important properties of eigen vectors? [5]

5. (a) Obtain the spin matrices σ_x ,σ_y and σ_z. What are the Assumptions involved in obtaining these matrices? Given: eigen vectors of i) σ_z are $|u >= \begin{pmatrix}1\\0\end{pmatrix}$, $|d >= \begin{pmatrix}0\\1\end{pmatrix}$, ii) σ_x are $|right >= \frac{1}{\sqrt{2}}(|u > +|d >)$, $|left >= \frac{1}{\sqrt{2}}(|u > -|d >)$, iii) σ_y are $|in >= \frac{1}{\sqrt{2}}(|u > +i|d >)$ and $|out >= \frac{1}{\sqrt{2}}(|u > -i|d >)$ each having eigen values of ± 1 [7]
   (b) Write a note on unitary transformation and its importance in time evolution of a quantum system. [3]

6. (a) Prove that $[b, b^\dagger] = 1$ and hence demonstrate that the Hamiltonian $H = \frac{\hbar\omega}{2}(bb^\dagger + b^\dagger b)$ can be reduced to $H = \hbar\omega(bb^\dagger + 1/2)$ [7]
   (b) Why is $b^\dagger$ and b operators are called as raising and lowering operators? [3]

   Given:-$b = (\frac{m\omega}{2\hbar})^{1/2}(x + \frac{ip}{m\omega})$, $b^\dagger = (\frac{m\omega}{2\hbar})^{1/2}(x - \frac{ip}{m\omega})$ where x, p are the position and momentum operators respectively.

7. (a) Show that $[L_x, L_y] = i\hbar L_z$ [5]

(b) Prove that $J_+|jm>=\sqrt{j(j+1)-m(m+1)}\,\hbar|jm+1>$ [5]

Given:-$[J_x, J_y] = i\hbar J_z$, $|jm>$ is an eigen state of the operator J^2 with an eigen value $j(j+1)$

PART B

Answer any **FOUR** of the following questions. Each question carries 5 marks. $[4 \times 5 = 20]$

8. Define Kroneckar delta. Explain the orthonormal property of the wave function.

9. An electron in a 2D infinite potential well needs to absorb an electromagnetic wave with a wavelength of $3500\ \mathrm{nm}$ to be excited from the lowest state to the next higher energy state. What is the length of the box if this is a square well potential well?

10. Construct spherical Newmann functions $n_1(x)$ and $n_2(x)$. Expand the sines and cosines to obtain approximate formulas for $n_1(x)$ and $n_2(x)$ valid when $x \ll 1$, and confirm that they blow up at the origin.

11. If $|\psi>= c_1(i|\alpha> +2|\beta>)$ and $|\phi>= c_2(|\alpha> -i|\beta>)$, find the values of c_1 and c_2 such that $|\psi>$ and $|\phi>$ are normalized. Assume that $|\alpha>$ and $|\beta>)$ are ortho-normal to each other.

12. If the Hamiltonian $H = \frac{P^2}{2m} + \frac{1}{2}kX^2$, find the value of $\frac{d<P>}{dt}$. Here, P and X are the momentum and the position operators respectively.

13. If $j = \frac{1}{2}$, what are the possible values of m?. Find the matrix form of J_+ and J_- operators.