Register Number: DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - IV SEMESTER

SEMESTER EXAMINATION: APRIL 2022

(Examination conducted in July 2022)

PH 8321/8320/8318/8315 – STATISTICAL PHYSICS

Time-2 1/2 hrs. **Maximum Marks-70**

This question paper has 3 printed pages and 2 parts

PART A

Answer any FIVE full questions. **(5x10=50)**

1. A box is completely thermally isolated from the external world. The box has two partitions labeled A and B having energies E_A and E_B respectively, associated with each other such that the composite system (the main box) has a total energy of $E=E_A+E_B$. The two partitions A and B interact thermally (to begin with both A and B are separately in thermal equilibrium but with differing temperatures with respect to each other) and attain equilibrium after a time τ. Show that as the system approach equilibrium, there is a net increase in the entropy of the composite system.
2. For a Canonical System the Partition Function is, in general, dependent on the parameter β and the external parameters x ; i.e. : $Z=Z(\beta,x)$. Given that for such a system the average energy is given as: $\bar{E}=-\frac{1}{Z}\frac{\partial Z}{\partial \beta}$ and that the macroscopic work done ***by*** the system is given by $đW=\frac{1}{\beta Z}\frac{\partial Z}{\partial x}$. Show that the laws of thermodynamics for the system leads to the expression: $S\equiv k_B(\ln Z+\beta\bar{E})$ where S is the entropy of the system.
3. Consider a mono-atomic gas having N atoms occupying a container of volume V. Given that the individual partition function of a single atom is given as: $\zeta=\frac{V}{h_0^3}\int_{-\infty}^{+\infty} e^{-\frac{\beta}{2m}\vec{p}^2}\, d^3\vec{\mathbf{p}}$:
   (a) Using the fact that a Gaussian integral evaluates to $\sqrt{\pi}$, compute the expression for ζ.
   (b) Obtain the partition function Z for the entire gas.

(c) Evaluate the mean pressure: $\bar{P}=\frac{1}{\beta}\frac{\partial \ln Z}{\partial V}$ and from this, the equation of state of the mono-atomic gas. [3+2+5]

4. Consider a system of N identical particles confined to a volume V and having a temperature T . We can, in addition, assume the system to be characterized by a chemical potential μ . Assume this system to be in a macroscopic state referenced by E_R and contributed to by each microstate of energy ε_r containing n_r particles, such that the partition function for such a Grand Canonical Distribution is given as: $Z_{\text{GC}}=\left(\sum_{n_1} e^{-\beta n_1(\varepsilon-\mu)}\right)\left(\sum_{n_2} e^{-\beta n_2(\varepsilon-\mu)}\right)\ldots$. Using quantum statistics (both Fermi-Dirac and Bose-Einstein), it is seen that the mean occupancy in each state is $\bar{n}_r=-\frac{1}{\beta}\frac{\partial \ln Z_{\text{GC}}}{\partial \varepsilon_r}=\frac{1}{e^{\beta(\varepsilon_r-\mu)}\pm 1}$. Obtain the Boltzmann Limit to this distribution when: Condition 1: $\beta(\varepsilon_r-\mu)\gg 1$ and Condition 2: $\bar{n}_r\ll 1$ are satisfied.
5. For radiation in an enclosure at temperature T , from general thermodynamical arguments show that the mean photon number density is a function of temperature alone and, further, does not depend on the shape of the enclosure.
6. Explain Einstein's Derivation of the Planck Radiation Law.
7. Obtain the expression for the mean energy of a Fermion Gas. You may use the general result that: $\int_0^\infty \frac{\varphi(\varepsilon)}{e^{\beta(\varepsilon-\mu)}-1}\, d\varepsilon=\int_0^\mu \varphi(\varepsilon)\, d\varepsilon+\frac{\pi^2}{6}\frac{1}{\beta^2}\left(\frac{d\varphi}{d\varepsilon}\right)\Big|_{\varepsilon=\mu}+\cdots$.

PART B

Answer any <u>FOUR</u> full questions. **(4x5=20)**

8. A box is completely thermally isolated from the external world. The box has two partitions labeled A and B containing two different ideal gases each in equilibrium but at different temperatures. Box A is estimated to have an external parameter such that it is in a macrostate having energy E_A , while box B has the macrostate associated with the same parameter with an energy E_B . We know that $E_A > E_B$. The systems A and B are allowed to thermally interact and eventually after a time τ attain equilibrium. Making the same assumptions as for the Kinetic Theory of Gases, express the final rms velocities of the particles in boxes A and B in terms of E_A and E_B (after they attain equilibrium).
9. A system approximated to a Simple Harmonic Oscillator and having energy levels given by $E_n=\left(n+\frac{1}{2}\right)\hbar\omega$ (with ω being the characteristic frequency of the oscillator) is assumed to be brought in contact with a thermal bath (or heat reservoir) of temperature T (which is low enough so that: $k_{\text{B}}T\ll\frac{\hbar^2\pi^2}{2mL^2}$). Compute the ratio of the probability of the system being in the second excited state ($n=2$) to the probability of it being in the ground state (the ground state corresponds to $n=0$).

10. A classical ideal gas is encased in an infinite insulated cylinder placed vertically. The force due to gravity acts along the length of this cylinder. Compute:

(a) The average kinetic energy of the molecules of gas in the cylinder

(b) The average potential energy of the particles (molecules). [2+3]

11. Compute the rms velocity of Oxygen (O_2) molecules at room temperature (absolute value of 300 K). The mass of Oxygen molecule is 32 u .

12. Consider a 3 particle system whose microstates are described by Simple Harmonic Oscillator energy levels with $\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega$ with ω being the characteristic oscillation frequency of the system. Compute the partition function if only the first 3 states are available to the particles and if the particles are assumed to be Fermions (remember that for the SHO system, $n = 0,1,2,3,\ldots$).

13. Assume that each particle in an ideal Fermi Gas obtains its momentum due to the uncertainty principle: $x\, p \sim \hbar$. Given that the density is defined as $\rho = \frac{m}{\frac{4\pi}{3} x^3}$ and that pressure may be approximated as $P \sim n\, p\, v$ with $n = \frac{\rho}{m}$ defined as the number density and $v = \frac{p}{m}$ being the velocity, obtain an approximate equation of state for the ideal Fermi Gas in the ***relativistic*** limit.