Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE – 27

M.Sc. STATISTICS – II SEMESTER

SEMESTER EXAMINATION – JULY 2022

ST 8121: Distribution Theory

Time: 2½ hrs **Max: 70 Marks**

This question paper has **TWO** printed pages and **TWO** sections

SECTION – A

I Answer any SIX of the following: 6x 3= 18

1. State and prove a memoryless property of exponential distribution.
2. For a random variable X with the following probability density function, find the mean and variance.
$$f(x) = \frac{x}{\alpha^2} e^{-\frac{x^2}{2\alpha^2}}, \qquad \alpha > 0, x > 0.$$
3. For any two random variables X and Y, prove that $EE(X|Y)) = E(X)$.
4. Define multinomial distribution. Give an example.
5. For a vector random variable Y with $E(Y) = \mu$, $V(Y) = \Sigma$ and square matrix A, show that $E(Y^T AY) = trace(A\Sigma) + \mu^T A\mu$.
6. State the pdf of the non-central F distribution. Mention its mean and variance.
7. Define order statistic and obtain the distribution function of first-order statistic.
8. If $X_1, \dots, X_n$ be independent random variables from a distribution with pdf
$$f(x) = \alpha \frac{k^\alpha}{x^{\alpha+1}}, \qquad x \geq k, k > 0\ \alpha > 0.$$
Find the distribution of $X_{(1)} = min\,(X_1, \dots, X_n)$.

SECTION – B

II Answer any FOUR of the following: 4 x 13 = 52

9. A) If X follows exponential with rate λ, show that Y=$[X]$ follows geometric distribution with parameter $1 - e^{-\lambda}$. .

B) Define negative binomial distribution. Find its mean and variance

C) Define standard normal distribution. For a standard normal distribution Z show that $E(Z^{n+1}) = nE(Z^{n-1}), n$ is a non-negative integer, and hence show that all odd ordered moments are zero. (3+5+5)

10. A) Define Weibull distribution. Find the distribution function and quantile function of Weibull distribution.
B) Find the mean and variance of the truncated Poisson distribution (truncated at zero)
C) Explain how to find the probability distribution of the range. (5+4+4)

11. A) Define multivariate normal random variable. Find the MGF of the same.
B) Let X and Y are two random variables with following joint probability density function (pdf)

$$f(x,y) = \begin{cases} \frac{1+xy}{4}, & 0 < x < 1,\ 0 < y < 1 \\ 0 & otheriwse \end{cases}$$

Find E (Y|X=0.5) (8+5)

12. A) If X and Y are independently distributed as gamma with parameters (α, θ) and (β, θ) respectively. Find the probability distribution of $U = \frac{X}{X+Y}$ and $V = X + Y$.
Check whether U and V are independent.
B) If X follows t distribution with n degrees of freedom, find the probability distribution of X^2. (8+5)

13. A) Define non-central chi-square distribution and obtain its MGF.
B) If $Y \sim N_p(\mu, \Sigma)$ show that $(Y-\mu)^T \Sigma^{-1} (Y-\mu) \sim \chi^2(p)$. (7+6)

14. A) Let $X_1, \dots, X_n$ be a random sample from a continuous distribution. Derive the probability density function of r^{th} order statistic.
B) If $X_1, \dots, X_n$ be a random sample from U (0, 1), find the probability distribution of r^{th} order statistic.
C) If $X_1, \dots, X_n$ are iid random variables from a pdf $f(x)$, derive the joint probability density function of r^{th} and s^{th} order statistics Y_r and Y_s ,where $1 \le r < s \le n$. (5+2+6)