Date:

Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.Sc. STATISTICS - II SEMESTER
SEMESTER END EXAMINATION: APRIL 2022
(Examination conducted in July 2022)

ST 8221 – Testing of hypothesis & Interval Estimation

Time- 2 ½ hrs Max Marks -70

This question paper contains two printed pages and two parts.

Part A

I. Answer any 6 questions. 3x6=18

1. Distinguish between size of a test and level of significance.
2. Explain the terms (i) Randomized and Non-randomized tests. (ii) P- value.
3. What is a uniformly most powerful unbiased test? Explain its need.
4. Define likelihood ratio test. Mention any two properties of it.
5. State Neyman-Pearson generalized lemma. Explain its use.
6. Describe the Pivotal quantity method of constructing confidence interval.
7. Derive the 100(1-α)% confidence intervals for mean of normal population when variance is unknown.
8. Define the following: (i) Confidence sets. (ii) UMA Confidence interval (iii) one-sided confidence interval.

Part B

II. Answer any 4 questions. 13x4=52

9. a. State and prove Neyman Pearson Lemma. (8)
b. Define most powerful test. Establish the relation between Type II error and power of a test. (5)

10. a. Construct a Uniformly Most powerful test for testing H_0: $\theta \geq \theta_0$ against H_1: $\theta < \theta_0$ where θ is the mean of exponential distribution. (7)
b. Develop a MP test for testing H_0: $\theta = \theta_0$ against H_1: $\theta = \theta_1\ (> \theta_0)$ where θ is the parameter of $U(0, \theta)$ distribution. (6)

11. a. Derive a LR test procedure for testing H_0: $\mu = \mu_0$ against H_1: $\mu \neq \mu_0$ where μ is the mean of normal distribution with unknown variance. (9)
b. Examine whether normal distribution with mean zero and variance σ^2 possess MLR property. (4)

12. a. Under Cramer-Rao regularity conditions, prove that $-2\log\lambda$ follows chi-square distribution with 1 degree of freedom. (6)

b. Establish the non-existence of UMP test for testing simple null hypothesis against two sided alternatives in one parameter exponential family. (7)

13. a. Explain Pearson's chi-square test for goodness of fit stating necessary assumptions. (7)

b. Write a short note on Wald's sequential probability ratio test. (6)

14. a. Let $x_1, x, \ldots, x_n$ be a random sample from a probability distribution with density

$$f(x) = (\theta + 1)x^{\theta}, \quad 0 < x < 1.$$

Obtain $100(1-\alpha)\%$ Uniformly Most Accurate lower bound for θ. (6)

b. Let $x_1, x_2, \ldots, x_n$ be a random sample from Uniform distribution over $(0, \theta)$. Find the shortest expected length confidence interval for θ. (7)

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