Part-B

21. One gram of salt is dissolved in water that is filled to a height of 5 cm in a beaker of diameter 10 cm. The accuracy of length measurement is 0.01 cm while that of mass measurement is 0.01 mg. When measuring the concentration C, the fractional error $\Delta C/C$ is
(a) 0.8% (b) 0.14% (c) 0.5% (d) 0.28%

22. A system can have three energy levels: $E = 0, \pm\varepsilon$. The level E = 0 is doubly degenerate, while the others are non-degenerate. The average energy at inverse temperature β is
(a) $-\varepsilon \tanh(\beta\varepsilon)$ (b) $\dfrac{\varepsilon\left(e^{\beta\varepsilon} - e^{-\beta\varepsilon}\right)}{\left(1 + e^{\beta\varepsilon} + e^{-\beta\varepsilon}\right)}$ (c) Zero (d) $-\varepsilon \tanh\left(\dfrac{\beta\varepsilon}{2}\right)$

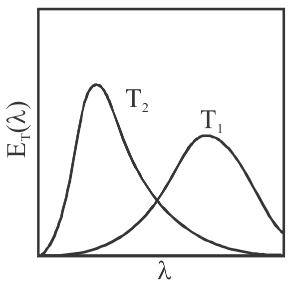
23. For a particular thermodynamics system the entropy S is related to the internal energy U and volume V by
$$S = c\,U^{3/4}\,V^{1/4}$$
where c is a constant. The Gibbs potential $G = U - TS + pV$ for this system is
(a) $\dfrac{3pU}{4T}$ (b) $\dfrac{cU}{3}$ (c) zero (d) $\dfrac{US}{4V}$

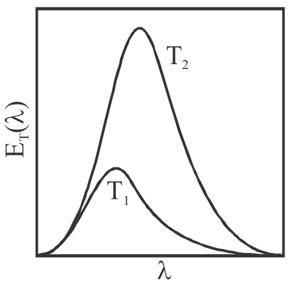
24. An op-amp based voltage follower
(a) is useful for converting a low impedance source into a high impedance source
(b) is useful for converting a high impedance source into a low impedance source
(c) has infinitely high closed loop output impedance
(d) has infinitely high closed loop gain

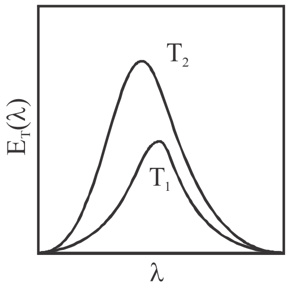
25. A particle of mass m in three dimensions is in the potential
$$V(r) = \begin{cases} 0 & r < a \\ \infty & r \geq a \end{cases}$$

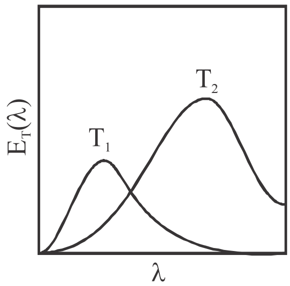
Its ground state energy is
(a) $\dfrac{\pi^2\hbar^2}{2ma^2}$ (b) $\dfrac{\pi^2\hbar^2}{ma^2}$ (c) $\dfrac{3\pi^2\hbar^2}{2ma^2}$ (d) $\dfrac{9\pi^2\hbar^2}{2ma^2}$

26. Which of the graphs below gives the correct qualitative behavior of the energy density $E_T(\lambda)$ of blackbody radiation of wavelength λ at two temperatures T_1 and T_2 ($T_1 < T_2$)?

(a)


(b)


(c) 

(d) 

27. Given that $\hat{p}_r = -i\hbar\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)$, the uncertainty Δp_r in the ground state

$$\psi_0(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

of the hydrogen atom is

(a) $\frac{\hbar}{a_0}$ (b) $\frac{\sqrt{2}\hbar}{a_0}$ (c) $\frac{\hbar}{2a_0}$ (d) $\frac{2\hbar}{a_0}$

28. An RC network produces a phase-shift of 30º. How many such RC networks should be cascaded together and connected to a Common Emitter amplifier so that the final circuit behaves as an oscillator?

(a) 6 (b) 12 (c) 9 (d) 3

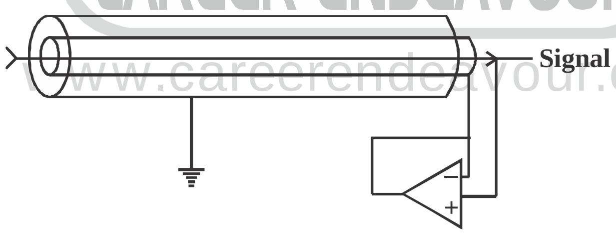
29. The free energy F of a system depends on a thermodynamics variable ψ as

$$F = -\alpha\psi^2 + b\psi^6$$

with a, b > 0. The value of ψ, when the system is in thermodynamic equilibrium, is

(a) zero (b) $\pm(a/6b)^{1/4}$ (c) $\pm(a/3b)^{1/4}$ (d) $\pm(a/b)^{1/4}$

30. The inner shield of a triaxial conductor is driven by an (ideal) op-amp follower circuit as shown. The effective capacitance between the signal-carrying conductor and ground is



(a) unaffected (b) doubled (c) halved (d) made zero

31. Consider a system of two non-interacting identical fermions, each of mass *m* in an infinite square well potential of width *a*. (Take the potential inside the well to be zero and ignore spin). The composite wavefunction for the system with total energy

$$E = \frac{5\pi^2\hbar^2}{2ma^2}$$

is

(a) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) - \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(b) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)+\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(c) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{2a}\right)-\sin\left(\frac{3\pi x_1}{2a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(d) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\cos\left(\frac{\pi x_2}{a}\right)-\sin\left(\frac{\pi x_2}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

32. A particle of mass m in the potential $V(x,y)=\frac{1}{2}m\omega^2\left(4x^2+y^2\right)$, is in an eigenstate of energy $E=\frac{5}{2}\hbar\omega$. The corresponding un-normalized eigenfunction is

(a) $y\exp\left[-\frac{m\omega}{2\hbar}\left(2x^2+y^2\right)\right]$ (b) $x\exp\left[-\frac{m\omega}{2\hbar}\left(2x^2+y^2\right)\right]$

(c) $y\exp\left[-\frac{m\omega}{2\hbar}\left(x^2+y^2\right)\right]$ (d) $xy\exp\left[-\frac{m\omega}{2\hbar}\left(x^2+y^2\right)\right]$

33. A particle of mass *m* and coordinate *q* has the Lagrangian

$$L=\frac{1}{2}m\dot{q}^2-\frac{\lambda}{2}q\dot{q}^2$$

where λ is a constant. The Hamiltonian for the system is given by

(a) $\frac{p^2}{2m}+\frac{\lambda qp^2}{2m^2}$ (b) $\frac{p^2}{2(m-\lambda q)}$ (c) $\frac{p^2}{2m}+\frac{\lambda qp^2}{2(m-\lambda q)^2}$ (d) $\frac{pq}{2}$

34. If $\vec{A}=yz\hat{i}+zx\hat{j}+xy\hat{k}$ and C is the circle of unit radius in the plane defined by z = 1, with the centre on the z-axis, then the value of the integral $\oint_C \vec{A}.\vec{d\ell}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

35. Given, $\sum_{n=0}^{\infty}P_n(x)t^n=\left(1-2xt+t^2\right)^{-1/2}$, for $|t|<1$, the value of $P_5(-1)$ is

(a) 0.26 (b) 1 (c) 0.5 (d) –1

36. A charged particle is at a distance *d* from an infinite conducting plane maintained at zero potential. When released from rest, the particle reaches a speed u at a distance d/2 from the plane. At what distance from the plane will the particle reach the speed 2u?

(a) d/6 (b) d/3 (c) d/4 (d) d/5

37. Consider the matrix

$$M=\begin{pmatrix}0 & 2i & 3i\\ -2i & 0 & 6i\\ -3i & -6i & 0\end{pmatrix}$$

The eigenvalues of M are

(a) –5, –2, 7 (b) –7, 0, 7 (c) –4i, 2i, 2i (d) 2, 3, 6

38. Consider the differential equation $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0$ with the initial conditions $x(0)=0$ and $\dot{x}(0)=1$. The solution $x(t)$ attains its maximum value when 't' is

(a) 1/2 (b) 1 (c) 2 (d) ∞

39. A light source is switched on and off at a constant frequency *f*. An observer moving with a velocity *u* with respect to the light source will observe the freuqency of the switching to be

(a) $f\left(1-\frac{u^2}{c^2}\right)^{-1}$ (b) $f\left(1-\frac{u^2}{c^2}\right)^{-1/2}$ (c) $f\left(1-\frac{u^2}{c^2}\right)$ (d) $f\left(1-\frac{u^2}{c^2}\right)^{1/2}$

40. If C is the contour defined by $|z|=\frac{1}{2}$, the value of the integral

$$\oint_C \frac{dz}{\sin^2 z}$$

is

(a) ∞ (b) $2\pi i$ (c) 0 (d) πi

41. The time period of a simple pendulum under the influence of the acceleration due to gravity g is T. The bob is subjected to an additional acceleration of magnitude $\sqrt{3}g$ in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of the bob will be

(a) 0º to the vertical and $\sqrt{3}T$ (b) 30º to the vertical and T/2

(c) 60º to the vertical and $T/\sqrt{2}$ (d) 0º to the vertical and $T/\sqrt{3}$

42. Consider an electromagnetic wave at the interface between two homogeneouos dielectric media of the dielectric constants ε_1 and ε_2. Assuming $\varepsilon_2>\varepsilon_1$ and non charges on the surface, the electric field vector $\vec{E}$ and the displacement vector $\vec{D}$ in the two media satisfy the following inequalities

(a) $|\vec{E}_2|>|\vec{E}_1|$ and $|\vec{D}_2|>|\vec{D}_1|$ (b) $|\vec{E}_2|<|\vec{E}_1|$ and $|\vec{D}_2|<|\vec{D}_1|$

(c) $|\vec{E}_2|<|\vec{E}_1|$ and $|\vec{D}_2|>|\vec{D}_1|$ (d) $|\vec{E}_2|>|\vec{E}_1|$ and $|\vec{D}_2|<|\vec{D}_1|$

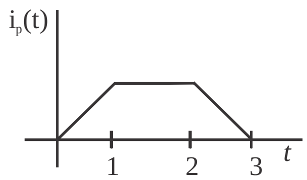
43. If the electrostatic potential in spherical polar coordinates is

$\varphi(r)=\varphi_0 e^{-r/r_0}$

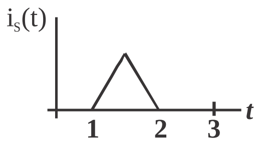
where φ_0 and r_0 are constants, then the charge density at a distance $r=r_0$ will be

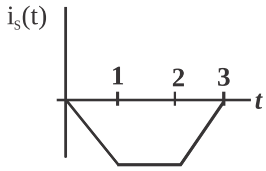
(a) $\frac{\varepsilon_0\varphi_0}{er_0^2}$ (b) $\frac{e\varepsilon_0\varphi_0}{2r_0^2}$ (c) $-\frac{\varepsilon_0\varphi_0}{er_0^2}$ (d) $-\frac{2e\varepsilon_0\varphi_0}{r_0^2}$

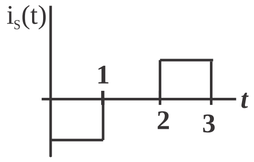
44. A current i_p flows through the primary coil of a transformer. The graph of $i_P(t)$ as a function of time 't' is shown in figure below

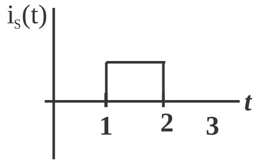


Which of the following graph represents the current i_S in the secondary coil?

(a) 

(b) 

(c) 

(d) 

45. A time-dependent current $\vec{I}(t) = Kt\hat{z}$ (where K is a constant) is switched on at $t = 0$ in an infinite current-carrying wire. The magnetic vector potential at a perpendicular distance 'a' from the wire is given (for time $t > a/c$) by

(a) $\hat{z}\dfrac{\mu_0 K}{4\pi c}\displaystyle\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$ (b) $\hat{z}\dfrac{\mu_0 K}{4\pi}\displaystyle\int_{-ct}^{ct} dz \frac{t}{\left(a^2+z^2\right)^{1/2}}$

(c) $\hat{z}\dfrac{\mu_0 K}{4\pi c}\displaystyle\int_{-ct}^{ct} dz \frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$ (d) $\hat{z}\dfrac{\mu_0 K}{4\pi}\displaystyle\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{t}{\left(a^2+z^2\right)^{1/2}}$

PART-C

46. The pressure of a non-relativistic free Fermi gas in three-dimensions depends, at T = 0, on the density of fermions *n* as

(a) $n^{5/3}$ (b) $n^{1/3}$ (c) $n^{2/3}$ (d) $n^{4/3}$

47. A doubel slit interference experiment uses a laser emitting light of two adjacent frequencies v_1 and v_2 $(v_1 < v_2)$. The minimum path difference between the interfering beams for which the interference pattern disappears is

(a) $\dfrac{c}{v_2+v_1}$ (b) $\dfrac{c}{v_2-v_1}$ (c) $\dfrac{c}{2(v_2-v_1)}$ (d) $\dfrac{c}{2(v_2+v_1)}$

48. The recently-discovered Higgs boson at hte LHC experiment has a decay mode into a photon and a Z boson. If the rest masses of the Higgs and Z boson are 125 GeV/c^2 and 90 GeV/c^2 respectively, and the decaying Higgs particle is at rest, the energy of the photon will approximately be

(a) $35\sqrt{3}$ *GeV* (b) 35 GeV (c) 30 GeV (d) 15 GeV

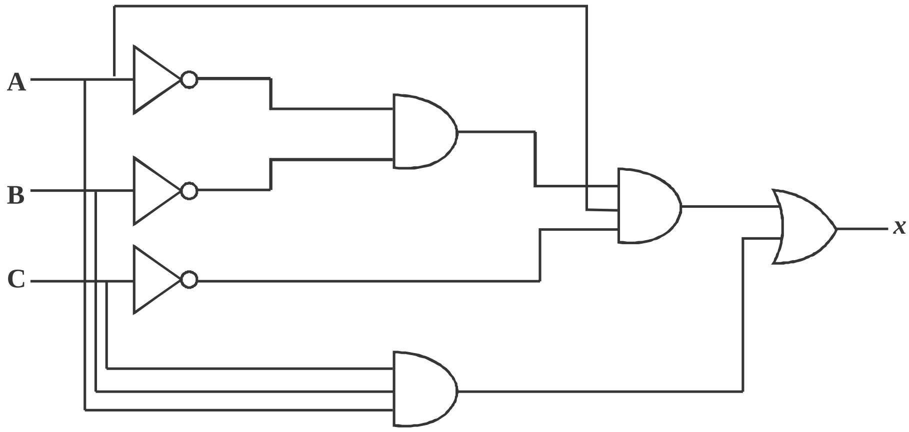
49. A permanently deformed even-even nucleus with $J^P = 2^+$ has rotational energy 93 keV. The energy of the next excited state is

(a) 372 keV (b) 310 keV (c) 273 keV (d) 186 keV

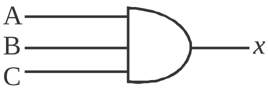
50. How much does the total angular momentum quantum number J change in the transition of Cr($3d^6$) atom as it ionizes to $Cr^{2+}\left(3d^4\right)$?

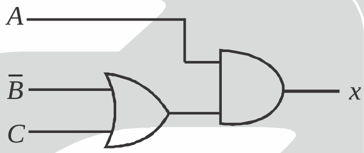
(a) increases by 2 (b) decreases by 2 (c) decreases by 4 (d) does not change

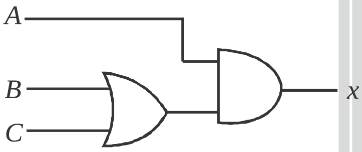
51. For the logic circuit shown in the figure below

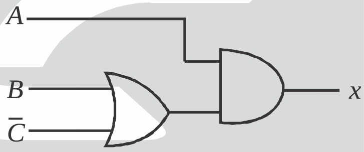


a simplified equivalent circuit is

(a) 

(b) 

(c) 

(d) 

52. A spectral line due to a transition from an electronic state p to an s state splits into three Zeeman lines in the presence of a strong magnetic field. At intermediate field strengths the number of spectral lines is

(a) 10 (b) 3 (c) 6 (d) 9

53. A particle in the infinite square well

$$V(x)=\begin{cases}0 & 0<x<a\\ \infty & \text{otherwise}\end{cases}$$

is prepared in a state with the wavefunction

$$\psi(x)=\begin{cases}A\sin^3\left(\frac{\pi x}{a}\right) & 0<x<a\\ 0 & \text{otherwise}\end{cases}$$

The expectation value of the energy of the particle is

(a) $\frac{5\hbar^2\pi^2}{2ma^2}$ (b) $\frac{9\hbar^2\pi^2}{2ma^2}$ (c) $\frac{9\hbar^2\pi^2}{10ma^2}$ (d) $\frac{\hbar^2\pi^2}{2ma^2}$

54. The average local internal magnetic field acting on an Ising spin is $H_{\text{int}}=\alpha M$, where M is the magnetization and α is a positive constant. At a temperature T sufficiently close to (and above) the critical temperature T_c, the magnetic susceptibility at zero external field is proportional to (k_B is the Boltzmann constant)

(a) $k_BT-\alpha$ (b) $(k_BT+\alpha)^{-1}$ (c) $(k_BT-\alpha)^{-1}$ (d) $\tanh(k_BT+\alpha)$

55. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps?
(a) 3/8 (b) 5/16 (c) 1/4 (d) 1/16

56. Consider an electron in a b.c.c. lattice with lattice constant a. A single particle wavefunction that satisfies the Bloch theorem will have the form $f(\vec{r})\exp(i\vec{k}.\vec{r})$, with $f(\vec{r})$ being

(a) $1+\cos\left[\frac{2\pi}{a}(x+y-z)\right]+\cos\left[\frac{2\pi}{a}(-x+y+z)\right]+\cos\left[\frac{2\pi}{a}(x-y+z)\right]$

(b) $1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$

(c) $1+\cos\left[\frac{\pi}{a}(x+y)\right]+\cos\left[\frac{\pi}{a}(y+z)\right]+\cos\left[\frac{\pi}{a}(z+x)\right]$

(d) $1+\cos\left[\frac{\pi}{a}(x+y-z)\right]+\cos\left[\frac{\pi}{a}(-x+y+z)\right]+\cos\left[\frac{\pi}{a}(x-y+z)\right]$

57. The dispersion relation for electrons in an f.c.c. crystal is given, in the tight binding approximation by

$$\varepsilon(k)=-4\varepsilon_0\left[\cos\frac{k_xa}{2}\cos\frac{k_ya}{2}+\cos\frac{k_ya}{2}\cos\frac{k_za}{2}+\cos\frac{k_za}{2}\cos\frac{k_xa}{2}\right]$$

where ‘a’ is the lattice constant and ε_0 is a constant with the dimension of energy. The x-component of the velocity of the electrons at $\left(\frac{\pi}{a},0,0\right)$ is

(a) $-\frac{2\varepsilon_0 a}{\hbar}$ (b) $\frac{2\varepsilon_0 a}{\hbar}$ (c) $-\frac{4\varepsilon_0 a}{\hbar}$ (d) $\frac{4\varepsilon_0 a}{\hbar}$

58. The following data is obtained in an expriment that measures the viscosity η as a function of molecular weight M for a set of polymers.

$M(Da)$	$\eta(kPa-s)$
990	0.28 ± 0.03
5032	30 ± 2
10191	250 ± 10
19825	2000 ± 200

The relation that best describes the dependence of η on M is
(a) $\eta\sim M^{4/9}$ (b) $\eta\sim M^{3/2}$ (c) $\eta\sim M^{2}$ (d) $\eta\sim M^{3}$

59. The integral $\int_0^1\sqrt{x}dx$ is to be evaluated up to 3 decimal places using Simpson’s 3-point rule. If the interval [0, 1] is divided into 4 equal parts, the correct result is
(a) 0.683 (b) 0.667 (c) 0.657 (d) 0.638

60. In a classical model, a scalar (spin-0) meson consists of a quark and an antiquark bound by a potential

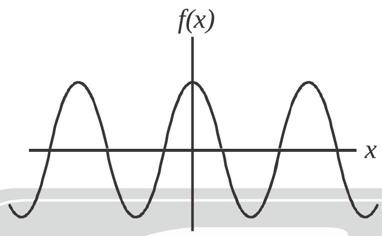
$$V(r)=ar+\frac{b}{r}$$

where a = 200 MeV fm^{-1} and b = 100 MeV fm. If the masses of the quark and antiquark are negligible, the mass of the meson can be estimated as approximately
(a) 141 MeV/c^2 (b) 283 MeV/c^2 (c) 353 MeV/c^2 (d) 425 MeV/c^2

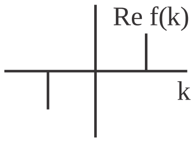
61. Let $y = \frac{1}{2}(x_1 + x_2) - \mu$, where x_1 and x_2 are independent and identically distributed Gaussian random variables of mean μ and standard deviation σ. Then $\frac{\langle y^4 \rangle}{\sigma^4}$ is

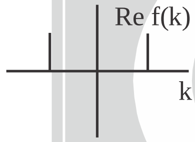
(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

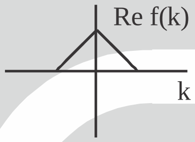
62. The graph of a real periodic function $f(x)$ for the range $[-\infty, \infty]$ is shown below

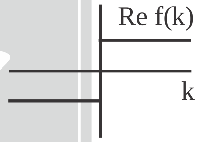


Which of the following graphs represents the real part of its Fourier transform?

(a) 

(b) 

(c) 

(d) 

63. The matrices

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

satisfy the commutation relations

(a) $[A,B] = B + C, [B,C] = 0, [C,A] = B + C$

(b) $[A,B] = C, [B,C] = A, [C,A] = B$

(c) $[A,B] = B, [B,C] = 0, [C,A] = A$

(d) $[A,B] = C, [B,C] = 0, [C,A] = B$

64. The function $\Phi(x,y,z,t) = \cos(z - vt) + \text{Re}(\sin(x + iy))$ satisfies the equation

(a) $\frac{1}{v^2}\frac{\partial^2 \Phi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi$

(b) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi$

(c) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\Phi$

(d) $\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)\Phi$

65. The coordinates and momenta $x_i, p_i\ (i=1,2,3)$ of a particle satisfy the canonical Poisson bracket relations $\{x_i, p_j\} = \delta_{ij}$. If $C_1 = x_2 p_3 + x_3 p_2$ and $C_2 = x_1 p_2 - x_2 p_1$ are constants of motion, and if $C_3 = \{C_1, C_2\} = x_1 p_3 + x_3 p_1$, then

(a) $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = C_2$
(b) $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = -C_2$
(c) $\{C_2, C_3\} = -C_1$ and $\{C_3, C_1\} = C_2$
(d) $\{C_2, C_3\} = C_1$ and $\{C_3, C_1\} = -C_2$

66. A canonical transformation relates the old coordinates (q, p) to the new ones (Q, P) by the relations $Q = q^2$ and $P = p/2q$. The corresonding time-independent generating function is

(a) $\frac{P}{q^2}$
(b) $q^2 P$
(c) q^2/P
(d) qP^2

67. The time evolution of a one-dimensional dynamical system is described by

$$\frac{dx}{dt} = -(x+1)\left(x^2 - b^2\right)$$

If this has one stable and two unstable fixed points, then the parameter 'b' satisfies

(a) 0 < b < 1
(b) b > 1
(c) b < –1
(d) b = 2

68. A charge (–e) is placed in vacuum at the point (d, 0, 0), where d > 0. The region $x \le 0$ is filled uniformly with a metal. The electric field at the point $\left(\frac{d}{2}, 0, 0\right)$ is

(a) $-\frac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$
(b) $\frac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$
(c) $\frac{e}{\pi\varepsilon_0 d^2}(1,0,0)$
(d) $-\frac{e}{\pi\varepsilon_0 d^2}(1,0,0)$

69. An electron is in the ground state of a hydrogen atom. The probability that it is within the Bohr radius is approximately equal to

(a) 0.60
(b) 0.90
(c) 0.16
(d) 0.32

70. A beam of light of frequency ω is reflected from a dielectric-metal interface at normal incience. The refractive index of the dielectric medium is n and that of the metal is $n_2 = n(1+i\rho)$. If the beam is polarised parallel to the interface, then the phase change experienced by the light upon reflection is

(a) $\tan\left(\frac{2}{\rho}\right)$
(b) $\tan^{-1}\left(\frac{1}{\rho}\right)$
(c) $\tan^{-1}\left(\frac{2}{\rho}\right)$
(d) $\tan^{-1}(2\rho)$

71. The scattering amplitude $f(\theta)$ for the potential $V(r) = \beta e^{-\mu r}$, where β and μ are positive constants, is given, in the Born approximation by

(in the following $b = 2k\sin\frac{\theta}{2}$ and $E = \frac{\hbar^2 k^2}{2m}$)

(a) $-\frac{4m\beta\mu}{\hbar^2\left(b^2+\mu^2\right)^2}$
(b) $-\frac{4m\beta\mu}{\hbar^2 b^2\left(b^2+\mu^2\right)}$
(c) $-\frac{4m\beta\mu}{\hbar^2\sqrt{b^2+\mu^2}}$
(d) $-\frac{4m\beta\mu}{\hbar^2\left(b^2+\mu^2\right)^3}$

72. The ground state eigenfunction for the potential $V(x) = -\delta(x)$, where $\delta(x)$ is the delta function, is given by $\psi(x) = Ae^{-\alpha|x|}$, where A and $\alpha > 0$ are constants. If a perturbation $H' = bx^2$ is applied, the first order correction to the energy of the ground state will be

(a) $\frac{b}{\sqrt{2}\alpha^2}$ (b) $\frac{b}{\alpha^2}$ (c) $\frac{2b}{\alpha^2}$ (d) $\frac{b}{2\alpha^2}$

73. A thin infinitely long solenoid placed along the z-axis contains a magnetic flux ϕ. Which of the following vector potentials corresponds to the magnetic field at an arbitrary point (x, y, z)?

(a) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2}, \frac{\phi}{2\pi}\frac{x}{x^2+y^2}, 0\right)$

(b) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2+z^2}, \frac{\phi}{2\pi}\frac{x}{x^2+y^2+z^2}, 0\right)$

(c) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{x+y}{x^2+y^2}, \frac{\phi}{2\pi}\frac{x+y}{x^2+y^2}, 0\right)$

(d) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{x}{x^2+y^2}, \frac{\phi}{2\pi}\frac{y}{x^2+y^2}, 0\right)$

74. The van der Waals equation of state for a gas is given by

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where, *P*, *V* and *T* represent the pressure, volume and temperature respectively, and *a* and *b* are constant parameters. At the critical point, where all the roots of the above cubic equation are degenerate, the volume is given by

(a) $\frac{a}{9b}$ (b) $\frac{a}{27b^2}$ (c) $\frac{8a}{27bR}$ (d) $3b$

75. An electromagnetically-shielded room is designed so that at a frequency $\omega = 10^7$ rad/s the intensity of the external radiation that penerates the room is 1% of the incident radiation. If $\sigma = \frac{1}{2\pi} \times 10^6 (\Omega m)^{-1}$ is the conductivity of the shielding material, its minimum thickness should be (given that ln10 = 2.3)

(a) 4.60 mm (b) 2.30 mm (c) 0.23 mm (d) 0.46 mm