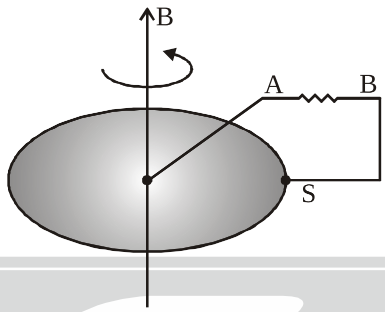
Part-B

21. A horizontal metal disc rotates about the vertical axis in a uniform magnetic field pointing up as shown in the figure. A circuit is made by connecting one end A of a resistor to the centre of the disc and the other end B to its edge through a sliding contact S. The current that flows through the resistor is



(a) zero (b) DC from A to B
(c) DC from B to A (d) AC

22. A spin $-\frac{1}{2}$ particle is in the state $\chi = \frac{1}{\sqrt{11}}\begin{pmatrix} 1+i \\ 3 \end{pmatrix}$ in the eigenbasis of S^2 and S_z. If we measure S_z the probabilities of getting $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively, are

(a) $\frac{1}{2}$ and $\frac{1}{2}$ (b) $\frac{2}{11}$ and $\frac{9}{11}$ (c) 0 and 1 (d) $\frac{1}{11}$ and $\frac{3}{11}$

23. Which of the following functions cannot be the real part of a complex analytic function of $z = x + iy$?

(a) $x^2 y$ (b) $x^2 - y^2$ (c) $x^3 - 3xy^2$ (d) $3x^2 y - y - y^3$

24. The motion of a particle of mass m in one dimension is described by the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 + \lambda x$. What is the difference between the (quantized) energies of the first two levels? (In the following, $\langle x \rangle$ is the expectation value of x in the ground state)

(a) $\hbar\omega - \lambda\langle x \rangle$ (b) $\hbar\omega + \lambda\langle x \rangle$ (c) $\hbar\omega + \frac{\lambda^2}{2m\omega^2}$ (d) $\hbar\omega$

25. Let ψ_{nlm} denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential $V(r)$. The expectation value of L_z in the state

$$\psi = \frac{1}{6}\left[\psi_{200} + \sqrt{5}\,\psi_{210} + \sqrt{10}\,\psi_{21-1} + \sqrt{20}\,\psi_{211}\right] \text{ is}$$

(a) $-\frac{5}{18}\hbar$ (b) $\frac{5}{6}\hbar$ (c) $\hbar$ (d) $\frac{5}{18}\hbar$

26. Three identical spin $-\frac{1}{2}$ fermions are to be distributed in two non-degenerate distinct energy levels. The number of ways this can be done is

(a) 8 (b) 4 (c) 3 (d) 2

27. Let A, B and C be functions of phase space variables (coordinates and momenta of a mechanical system). If {,} represents the Poisson bracket, the value of $\{A,\{B,C\}\} - \{\{A,B\},C\}$ is given by

(a) 0 (b) $\{B,\{C,A\}\}$ (c) $\{A,\{C,B\}\}$ (d) $\{\{C,A\},B\}$

28. If A, B and C are non-zero Hermitian operators, which of the following relations must be false?

(a) $[A, B] = C$ (b) $AB + BA = C$ (c) $ABA = C$ (d) $A + B = C$

29. The expression

$$\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\partial^2}{\partial x_4^2}\right)\frac{1}{(x_1^2+x_2^2+x_3^2+x_4^2)}$$ is proportional to

(a) $\delta(x_1+x_2+x_3+x_4)$ (b) $\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4)$

(c) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-3/2}$ (d) $(x_1^2+x_2^2+x_3^2+x_4^2)^{-2}$

30. Given that the integral $\int_0^\infty \frac{dx}{y^2+x^2} = \frac{\pi}{2y}$, the value of $\int_0^\infty \frac{dx}{(y^2+x^2)^2}$ is

(a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

31. The force between two long and parallel wires carrying currents I_1 and I_2 and separated by a distance D is proportional to

(a) $I_1 I_2 / D$ (b) $(I_1 + I_2) / D$ (c) $(I_1 I_2 / D)^2$ (d) $I_1 I_2 / D^2$

32. A loaded dice has the probabilities $\frac{1}{21},\frac{2}{21},\frac{3}{21},\frac{4}{21},\frac{5}{21}$ and $\frac{6}{21}$ of turning up 1, 2, 3, 4, 5 and 6, respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even?

(a) $\frac{144}{441}$ (b) $\frac{225}{441}$ (c) $\frac{221}{441}$ (d) $\frac{220}{441}$

33. A particle moves in a potential $V = x^2 + y^2 + \frac{z^2}{2}$. Which component (s) of the angular momentum is / are constant (s) of motion?

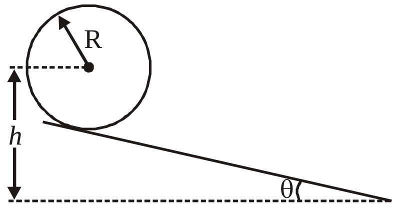
(a) none (b) L_x, L_y and L_z (c) only L_x and L_y (d) only L_z

34. The Hamiltonian of a relativistic particle of rest mass *m* and momentum *p* is given by $H = \sqrt{p^2+m^2} + V(x)$, in units in which the speed of light $c = 1$. The corresponding Lagrangian is

(a) $L = m\sqrt{1+\dot{x}^2} - V(x)$ (b) $L = -m\sqrt{1-\dot{x}^2} - V(x)$

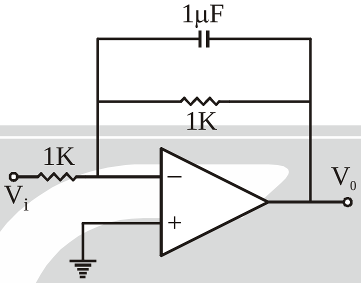
(c) $L = \sqrt{1+m\dot{x}^2} - V(x)$ (d) $L = \frac{1}{2}m\dot{x}^2 - V(x)$

35. A ring of mass m and radius R rolls (without slipping) down an inclined plane starting from rest. If the centre of the ring is initially at a height h, the angular velocity when the ring reaches the base is



(a) $\sqrt{g/(h-R)}\tan\theta$ (b) $\sqrt{g/(h-R)}$ (c) $\sqrt{g(h-R)/R^2}$ (d) $\sqrt{2g/(h-R)}$

36. Consider the op-amp circuit shown in the figure.



If the input is a sinusoidal wave $V_i = 5\sin(1000t)$, then the amplitude of the output V_0 is

(a) $\frac{5}{2}$ (b) 5 (c) $\frac{5\sqrt{2}}{2}$ (d) $5\sqrt{2}$

37. If one of the inputs of a J-K flip flop is high and the other is low, then the outputs Q and $\bar{Q}$

(a) oscillate between low and high in race-around condition
(b) toggle and the circuit acts like a T flip flop
(c) are opposite to the inputs
(d) follow the inputs and the circuit acts like an R-S flip flop

38. Two monochromatic sources, L_1, and L_2, emit light at 600 and 700 nm, respectively. If their frequency bandwidths are 10^{-1} and 10^{-3} GHz, respectively, then the ratio of linewidth of L_1 and L_2 is approximately

(a) 100 : 1 (b) 1 : 85 (c) 75 : 1 (d) 1 : 75

39. Let (V, A) and (V', A') denote two sets of scalar and vector potentials, and ψ a scalar function. Which of the following transformations leave the electric and magnetic fields (and hence Maxwell's equations) unchanged?

(a) $A' = A + \nabla\psi$ and $V = V - \frac{\partial\psi}{\partial t}$ (b) $A' = A - \nabla\psi$ and $V' = V + 2\frac{\partial\psi}{\partial t}$

(c) $A' = A + \nabla\psi$ and $V' = V + \frac{\partial\psi}{\partial t}$ (d) $A' = A - 2\nabla\psi$ and $V' = V - \frac{\partial\psi}{\partial t}$

40. Consider the melting transition of ice into water at constant pressure. Which of the following thermodynamic quantities does not exhibit a discontinuous change across the phase transition?

(a) internal energy (b) Helmholtz free energy
(c) Gibbs free energy (d) entropy

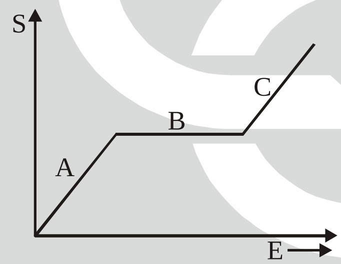
41. Two different thermodynamic systems are described by the following equations of state: $\frac{1}{T^{(1)}}=\frac{3RN^{(1)}}{2U^{(1)}}$ and $\frac{1}{T^{(2)}}=\frac{5RN^{(2)}}{2U^{(2)}}$ where $T^{(1,2)}, N^{(1,2)}$ and $U^{(1,2)}$ are respectively, the temperatures, the mole numbers and the internal energies of the two systems, and R is the gas constant. Let U_{tot} denote the total energy when these two systems are put in contact and attain thermal equilibrium. The ratio $\frac{U^{(1)}}{U_{tot}}$ is

(a) $\frac{5N^{(2)}}{3N^{(1)}+5N^{(2)}}$ (b) $\frac{3N^{(1)}}{3N^{(1)}+5N^{(2)}}$ (c) $\frac{N^{(1)}}{N^{(1)}+N^{(2)}}$ (d) $\frac{N^{(2)}}{N^{(1)}+N^{(2)}}$

42. The speed v of the molecules of mass m of an ideal gas obeys Maxwell's velocity distribution law at an equilibrium temperature T. Let (v_x, v_y, v_z) denote the components of the velocity and k_B the Boltzmann constant. The average value of $(\alpha v_x - \beta v_y)^2$, where α and β are constants, is

(a) $(\alpha^2-\beta^2)k_\beta T/m$ (b) $(\alpha^2+\beta^2)k_\beta T/m$ (c) $(\alpha+\beta)^2 k_\beta T/m$ (d) $(\alpha-\beta)^2 k_\beta T/m$

43. The entropy S of a thermodynamic system as a function of energy E is given by the following graph



The temperatures of the phases A, B and C, denoted by T_A, T_B and T_C, respectively, satisfy the following inequalities:

(a) $T_C > T_B > T_A$ (b) $T_A > T_C > T_B$ (c) $T_B > T_C > T_A$ (d) $T_B > T_A > T_C$

44. The physical phenomenon that cannot be used for memory storage applications is

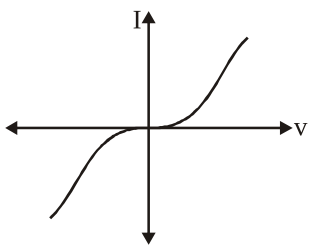
(a) large variation in magnetoresistance as a function of applied magnetic field

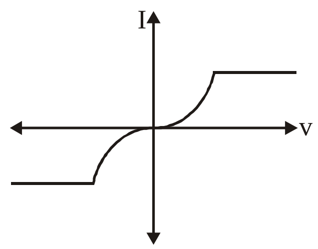
(b) variation in magnetization of a ferromagnet as a function of applied magnetic field

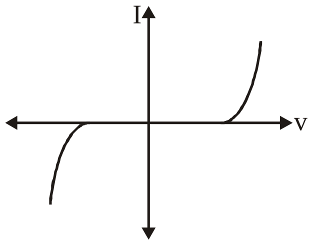
(c) variation in polarization of a ferroelectric as a function of applied electric field

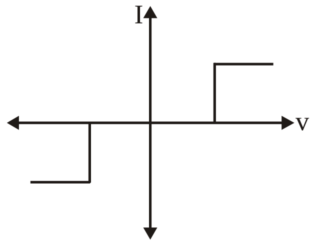
(d) variation in resistance of a metal as a function of applied electric field

45. Two identical Zener diodes are placed back to back in series and are connected to a variable DC power supply. The best representation of the I-V characteristics of the circuit is

(a)


(b)


(c) 

(d) 

Part-C

46. A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is

(a) $2\pi\sqrt{\frac{l+R}{g}}$ (b) $\frac{2\pi}{\sqrt{g}}(l^2+R^2)^{1/4}$

(c) $2\pi\sqrt{\frac{2R^2+2Rl+l^2}{g(R+l)}}$ (d) $\frac{2\pi}{\sqrt{g}}(2R^2+2Rl+l^2)^{1/4}$

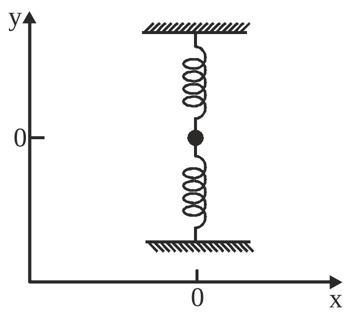
47. Spherical particles of a given material of density p are released from rest inside a liquid medium of lower density. The viscous drag force may be approximated by the Stoke's law, i.e, $F_d = 6\pi\eta R\text{v}$, where η is the viscosity of the medium, R the radius of a particle and v its instantaneous velocity. If $\tau(m)$ is the time taken by a particle of mass m to reach half its terminal velocity, then the ratio $\tau(8m)/\tau(m)$ is

(a) 8 (b) 1/8 (c) 4 (d) 1/4

48. A system of N classical non-interacting particles, each of mass m, is at a temperature T and is confined by the external potential $V(r)=\frac{1}{2}Ar^2$ (where A is a constant) in three dimensions. The internal energy of the system is

(a) $3Nk_BT$ (b) $\frac{3}{2}Nk_BT$ (c) $N(2mA)^{3/2}k_BT$ (d) $N\sqrt{\frac{A}{m}}\ln\left(\frac{k_BT}{m}\right)$

49. Consider a particle of mass m attached to two identical springs each of length l and spring constant k (see the figure below). The equilibrium configuration is the one where the springs are unstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, which of the following describes the equation of motion for small oscillations?



(a) $m\ddot{x}+\frac{kx^3}{l^2}=0$ (b) $m\ddot{x}+kx=0$ (c) $m\ddot{x}+2kx=0$ (d) $m\ddot{x}+\frac{kx^2}{l}=0$

50. If $\psi(x) = A\exp(-x^4)$ is the eigenfunction of a one dimensional Hamiltonian with eigenvalue E = 0, the potential $V(x)$ (in units where $\hbar = 2m = 1$) is

(a) $12x^2$ (b) $16x^6$ (c) $16x^6 + 12x^2$ (d) $16x^6 - 12x^2$

51. The electric field of an electromagnetic wave is given by $\vec{E} = E_0 \cos[\pi(0.3x + 0.4y - 1000t)]\hat{k}$. The associated magnetic field $\vec{B}$ is

(a) $10^{-3} E_0 \cos[\pi(0.3x + 0.4y - 1000t)]\hat{k}$

(b) $10^{-4} E_0 \cos[\pi(0.3x + 0.4y - 1000t)](4\hat{i} - 3\hat{j})$

(c) $E_0 \cos[\pi(0.3x + 0.4y - 1000t)](0.3\hat{i} + 0.4\hat{j})$

(d) $10^2 E_0 \cos[\pi(0.3x + 0.4y - 1000t)](3\hat{i} + 4\hat{j})$

52. The energy of an electron in a band as a function of its wave vector k is given by $E(k) = E_0 - B(\cos k_x a + \cos k_y a + \cos k_z a)$, where E_0, B and a are constants. The effective mass of the electron near the bottom of the band is

(a) $\frac{2\hbar^2}{3Ba^2}$ (b) $\frac{\hbar^2}{3Ba^2}$ (c) $\frac{\hbar^2}{2Ba^2}$ (d) $\frac{\hbar^2}{Ba^2}$

53. A DC voltage V is applied across a Josephson junction between two superconductors with a phase difference ϕ_0. If I_0 and k are constants that depend on the properties of the junction, the current flowing through it has the form

(a) $I_0 \sin\left(\frac{2eVt}{\hbar} + \phi_0\right)$ (b) $kV \sin\left(\frac{2eVt}{\hbar} + \phi_0\right)$ (c) $kV \sin\phi_0$ (d) $I_0 \sin\phi_0 + kV$

54. Consider the following ratios of the partial decay widths $R_1 = \frac{\Gamma(\rho^+ \to \pi^+ + \pi^0)}{\Gamma(\rho^- \to \pi^- + \pi^0)}$ and $R_2 = \frac{\Gamma(\Delta^{++} \to \pi^+ + p)}{\Gamma(\Delta^- \to \pi^- + n)}$. If the effects of electromagnetic and weak interactions are neglected, then R_1 and R_2 are, respectively,

(a) 1 and $\sqrt{2}$ (b) 1 and 2 (c) 2 and 1 (d) 1 and 1

55. The intrinsic electric dipole moment of a nucleus ${}^A_Z\text{X}$

(a) increases with Z, but independent of A (b) decreases with Z, but independent of A

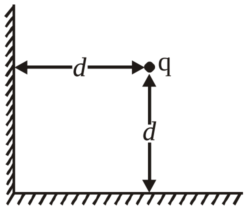
(c) is always zero (d) increases with Z and A

56. According to the shell model, the total angular momentum (in units of $\hbar$) and the parity of the ground state of the ${}^7_3\text{Li}$ nucleus is

(a) $\frac{3}{2}$ with negative parity (b) $\frac{3}{2}$ with positive parity

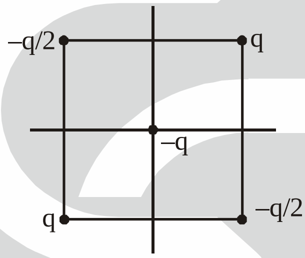
(c) $\frac{1}{2}$ with positive parity (d) $\frac{7}{2}$ with negative parity

57. A point charge q is placed symmetrically at a distance d from two perpendicularly placed grounded conducting infinite plates as shown in the figure. The net force on the charge (in units of $1/4\pi\in_0$) is



(a) $\frac{q^2}{8d^2}(2\sqrt{2}-1)$ away from the corner
(b) $\frac{q^2}{8d^2}(2\sqrt{2}-1)$ towards the corner
(c) $\frac{q^2}{2\sqrt{2}d^2}$ towards the corner
(d) $\frac{3q^2}{8d^2}$ away from the corner

58. Let four point charges q, –q/2, q and –q/2 be placed at the vertices of a square of side a. Let another point charge –q be placed at the centre of the square (see the figure).



Let $V(r)$ be the electrostatic potential at a point P at a distance $r >> a$ from the centre of the square. Then $V(2r)/V(r)$ is

(a) 1
(b) $\frac{1}{2}$
(c) $\frac{1}{4}$
(d) $\frac{1}{8}$

59. Let A and B be two vectors in three-dimensional Euclidean space. Under rotation, the tensor product $T_{ij} = A_i B_j$

(a) reduces to a direct sum of three 3-dimensional representations
(b) is an irreducible 9-dimensional representation
(c) reduces to a direct sum of a 1-dimensional, a 3-dimensional and a 5-dimensional irreducible representations
(d) reduces to a direct sum of a 1-dimensional and an 8-dimensional irreducible representation

60. Fourier transform of the derivative of the Dirac δ-function, namely $\delta'(x)$, is proportional to

(a) 0
(b) 1
(c) sink
(d) ik

61. A particle is in the ground state of an infinite square well potential given by,

$$V(x) = \begin{cases} 0 & \text{for } -a \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

The probability to find the particle in the interval between $-\frac{a}{2}$ and $\frac{a}{2}$ is

(a) $\frac{1}{2}$
(b) $\frac{1}{2}+\frac{1}{\pi}$
(c) $\frac{1}{2}-\frac{1}{\pi}$
(d) $\frac{1}{\pi}$

62. The expectation value of the x-component of the orbital angular momentum L_x in the state $\psi = \frac{1}{5}\left[3\psi_{2,1,-1} + \sqrt{5}\psi_{2,1,0} - \sqrt{11}\psi_{2,1,+1}\right]$ (where ψ_{nlm} are the eigenfunctions in usual notation), is

(a) $-\frac{\hbar\sqrt{10}}{25}(\sqrt{11}-3)$ (b) 0 (c) $\frac{\hbar\sqrt{10}}{25}(\sqrt{11}+3)$ (d) $\hbar\sqrt{2}$

63. A particle is prepared in a simultaneous eigenstate of L^2 and L_z. If $\ell(\ell+1)\hbar^2$ and $m\hbar$ are respectively the eigenvalues of L^2 and L_z, then the expectation value $\langle L_x^2 \rangle$ of the particle in this state satisfies

(a) $\langle L_x^2 \rangle = 0$

(b) $0 \le \langle L_x^2 \rangle \le \ell^2\hbar^2$

(c) $0 \le \langle L_x^2 \rangle \le \frac{\ell(\ell+1)\hbar^2}{3}$

(d) $\frac{\ell\hbar^2}{2} \le \langle L_x^2 \rangle \le \frac{\ell(\ell+1)\hbar^2}{2}$

64. If the electrostatic potential $V(r,\theta,\phi)$ in *a* charge free region has the form $V(r,\theta,\phi) = f(r)\cos\theta$, then the functional form of $f(r)$ (in the following *a* and *b* are constants) is

(a) $ar^2 + \frac{b}{r}$ (b) $ar + \frac{b}{r^2}$ (c) $ar + \frac{b}{r}$ (d) $a\ln\left(\frac{r}{b}\right)$

65. If $\mathbf{A} = \hat{i}yz + \hat{j}xz + \hat{k}xy$, then the integral $\oint_C A.dl$ (where C is along the perimeter of a rectangular area bounded by $x = 0$, $x = a$ and $y = 0$, $y = b$) is

(a) $\frac{1}{2}(a^3 + b^3)$ (b) $\pi(ab^2 + a^2b)$ (c) $\pi(a^3 + b^3)$ (d) 0

66. Consider an $n \times n (n > 1)$ matrix A, in which A_{ij} is the product of the indices i and j (namely $A_{ij} = ij$). The matrix A

(a) has one degenerate eigenvalue with degeneracy $(n - 1)$

(b) has two degenerate eigenvalues with degeneracies 2 and $(n - 2)$

(c) has one degenerate eigenvalue with degeneracy n

(d) does not have any degenerate eigenvalue

67. A child makes a random walk on a square lattice of lattice constant a taking a step in the north, east, south, or west directions with probabilities 0.255, 0.255, 0.245, and 0.245, respectively. After a large number of steps, N, the expected position of the child with respect to the starting point is at a distance

(a) $\sqrt{2} \times 10^{-2} Na$ in the north-east direction

(b) $\sqrt{2N} \times 10^{-2} a$ in the north-east direction

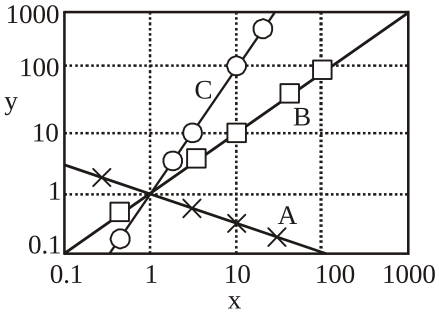
(c) $2\sqrt{2} \times 10^{-2} Na$ in the south-east direction

(d) 0

68. A Carnot cycle operates as a heat engine between two bodies of equal heat capacity until their temperatures become equal. If the initial temperatures of the bodies are T_1 and T_2, respectively, and $T_1 > T_2$ then their common final temperature is

(a) T_1^2 / T_2 (b) T_2^2 / T_1 (c) $\sqrt{T_1 T_2}$ (d) $\frac{1}{2}(T_1 + T_2)$

69. Three sets of data *A*, *B* and *C* from an experiment, represented by $\times$, $\square$ and $\bigcirc$, are plotted on a log-log scale. Each of these are fitted with straight lines as shown in the figure.



The functional dependence $y(x)$ for the sets A, B and C are, respectively

(a) $\sqrt{x}, x$ and x^2 (b) $-\frac{x}{2}, x$ and $2x$ (c) $\frac{1}{x^2}, x$ and x^2 (d) $\frac{1}{\sqrt{x}}, x$ and x^2

70. A sample of Si has electron and hole mobilities of 0.13 and 0.05 m^2/V-s respectively at 300K. It is doped with P and Al with doping densities of $1.5 \times 10^{21}/m^3$ and $2.5 \times 10^{21}/m^3$ respectively. The conductivity of the doped Si sample at 300K is

(a) $8\Omega^{-1}m^{-1}$ (b) $32\Omega^{-1}m^{-1}$ (c) $20.8\Omega^{-1}m^{-1}$ (d) $83.2\Omega^{-1}m^{-1}$

71. A 4-variable switching function is given by $f = \Sigma(5,7,8,10,13,15) + d(0,1,2)$, where *d* is the do-not-care-condition. The minimized form of *f* in sum of products (SOP) form is

(a) $\bar{A}\bar{C} + \bar{B}\bar{D}$ (b) $A\bar{B} + C\bar{D}$ (c) $AD + BC$ (d) $\bar{B}\bar{D} + BD$

72. A perturbation $V_{pert} = aL^2$ is added to the Hydrogen atom potential. The shift in the energy level of the 2P state, when the effects of spin are neglected up to second order in *a*, is

(a) 0 (b) $2a\hbar^2 + a^2\hbar^4$ (c) $2a\hbar^2$ (d) $a\hbar^2 + \frac{3}{2}a^2\hbar^4$

73. A gas laser cavity has been designed to operate at $\lambda = 0.5\mu m$ with a cavity length of 1m. With this set-up, the frequency is found to be larger than the desired frequency by 100 Hz. The change in the effective length of the cavity required to retune the laser is

(a) -0.334×10^{-12} m (b) 0.334×10^{-12} m
(c) 0.167×10^{-12} m (d) -0.167×10^{-12} m

74. The spectroscopic symbol for the ground state of $_{13}Al$ is $^2P_{1/2}$. Under the action of a strong magnetic field (when L-S coupling can be neglected) the ground state energy level will split into

(a) 3 levels (b) 4 levels (c) 5 levels (d) 6 levels

75. A uniform linear monoatomic chain is modeled by a spring-mass system of masses *m* separated by nearest neighbor distance *a* and spring constant $m\omega_0^2$. The dispersion relation for this system is

(a) $\omega(k) = 2\omega_0\left(1-\cos\left(\frac{ka}{2}\right)\right)$ (b) $\omega(k) = 2\omega_0 \sin^2\left(\frac{ka}{2}\right)$

(c) $\omega(k) = 2\omega_0 \sin\left(\frac{ka}{2}\right)$ (d) $\omega(k) = 2\omega_0 \tan\left(\frac{ka}{2}\right)$