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ST. JOSEPH'S COLLEGE (AUTONOMOUS) BANGALORE-27
MID SEMESTER TEST- AUGUST 2016
M.Sc. MATHEMATICS – I SEMESTER
MT 7414: ORDINARY DIFFERENTIAL EQUATIONS

Time: 1 ½ hours **Max. Marks: 35**

Answer any FIVE of the following questions. **5X7=35**

1. If y_1 and y_2 are two solutions of $y'' + a(x)y = 0$ then show that $z = y_1 y_2$ is a solution of $z''' + 4a(x)z' + 2a'(x)z = 0$. Also show that if $\{\phi_1, \phi_2\}$ forms a fundamental set of $y'' + a(x)y = 0$ then $\{\phi_1^2, \phi_1\phi_2, \phi_2^2\}$ is a fundamental set of $z''' + 4a(x)z' + 2a'(x)z = 0$. (7)

2. Let $\{\phi_j(x), j = 1 \text{ to } n\}$ be the fundamental set for $L_n(y) = 0$ defined over the interval I then the Wronskian $W\{\phi_j(x), j = 1 \text{ to } n\} = W\{\phi_j(x_0), j = 1 \text{ to } n\} e^{-\int_{x_0}^{x} \frac{a_1(t)}{a_0(t)} dt}$ where $x_0, x \in I$ and $a_0(x)$, $a_1(x)$ are the coefficients of $y^{(n)}$ and $y^{(n-1)}$ respectively. (7)

3. (i) Find the adjoint differential equation of $x^2 y'' + (2x^3 + 7x)y' + (8x^2 + 8)y = 0$. (4)

   (ii) Determine whether $y'' - \tan x\, y' + y = 0$ is self adjoint, if not transform the given differential equation into an equivalent self adjoint differential equation. (3)

4. If $f(x)$ and $g(x)$ be two n times differentiable function defined over an interval $[a, b]$ then prove that $\int_a^b g(x) L_n f(x)\, dx - \int_a^b f(x) L_n^* g(x)\, dx = [f, g](b) - [f, g](a)$ (7)

5. Solve $y'' - 3y' + 2y = \frac{1}{1 + e^{-x}}$ by method of variation of parameters. (7)

6. Let $\phi_1(x)$ be a solution of $\frac{d}{dx}(p(x)y') + q_1(x)y = 0$ having consecutive zeros at $x = a$ and $x = b$, $a < b$. Let $\phi_2(x)$ be a solution of $\frac{d}{dx}(p(x)y') + q_2(x)y = 0$ in the interval $[a, b]$ where $q_2(x) > q_1(x)$ in $[a, b]$ then $\phi_2(x)$ has atleast one zero at some point of the open interval $a < x < b$. (7)

7. Obtain the series solution of $4xy'' + 2y' + y = 0$ using Frobenius method about the regular singular point $x = 0$. (7)