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ST.JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
MID SEMESTER TEST- AUGUST 2016
M.Sc. MATHEMATICS-III SEMESTER
MT 9115 - COMPUTATIONAL LINEAR ALGEBRA

TIME: 1 ½ HOURS MAX. MARKS: 35

Answer any FIVE of the following questions. 5 X 7 = 35

1. Prove that if A is an algebra, with unit element over F, then A is isomorphic to a subalgebra of $A(V)$ for some vector space V over F.
2. Prove that if V is finite dimensional vector space over F, then $T \in A(V)$ is regular if and only if T maps V onto itself.
3. Prove that if $\lambda \in F$ is characteristic root of T and $T \in A(V)$, then for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.
4. a) Prove that if V is finite dimensional vector space over F, then $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0.

   b) Show that $(2,-5,3) \in V_3(R) \notin L[S]$, where $S = \{(1,-3,2),(2,-4,-1),(1,-5,7)\}$.
5. a) Define matrices of a linear transformation.

   b) Find the matrix of the liner transformation corresponding to $(1, 1+x, 1+x^2, \ldots\ldots, 1+x^{n-1})$
6. If V is n-dimensional vector space over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $v_1, v_2, \_\_\_\_, v_n$ and the matrix $m_2(T)$ in the basis $w_1, w_2, \_\_\_\_, w_n$ of V over F, then prove that there is an element $C \in F_n$ such that $m_2(T) = Cm_1(T)C^{-1}$.
7. If V is a vector space over a field F, then prove that the double dual of V is isomorphic to V.