## St. Joseph's College, Autonomous, Bangalore
### B.Sc-I Semester
Mid semester Examination: August, 2018
### MT 118: Mathematics

**Duration:** 1 Hour                                          **Max. Marks:** 30

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(1) The paper contains two pages.

(2) Attempt any **SIX** questions.

(3) Each question carries 5 marks.

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(1) Find the rank of following matrix by reducing it to normal form          [5 marks]

$$A = \begin{pmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{pmatrix}$$

(2) (a) Find the $n^{\text{th}}$ derivative of   $\sin(bx + c)$.          [2 marks]

   (b) Find the $n^{\text{th}}$ derivative of   $\dfrac{x^2}{(x-1)^2(x-2)}$.          [3 marks]

(3) State and prove Leibnitz Theorem.          [5 marks]

(4) Find the equation of the plane that bisects the obtuse angle between the planes $x + 2y - 2z + 1 = 0$ and $2x + y - 2z = 0$.          [5 marks]

(5) Find the real values of $\lambda$ for which the system of linear equations

$$x + 2y + 3z = \lambda x$$

$$3x + y + 2x = \lambda y$$

$$2x + 3y + z = \lambda z$$

has non-zero solutions.          [5 marks]

(6) If $y = \log(x + \sqrt{1 + x^2})$ prove that          [5 marks]

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2 y_n = 0.$$

(7) (a) Find the equation of the plane through the points $(1,0,-1)$ , $(3,2,2)$ and parallel to the line $\dfrac{x-1}{3} = \dfrac{y-1}{-2} = \dfrac{z-2}{3}$. [2 marks]

(b) Show that the lines $\dfrac{x-3}{3} = \dfrac{y-2}{-4} = \dfrac{z+1}{1}$ and $x+2y+3z = 0 = 2x+4y+3z+3$ are coplanar. [3 marks]

(8) If $u = \dfrac{1}{x^2+y^2+z^2}$ show that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial x^z} = 0.$ [5 marks]