



Registration Number:

Date & Session

ST. JOSEPH'S UNIVERSITY, BENGALURU-27
M. Sc (BIG DATA ANALYTICS) - I SEMESTER
SEMESTER EXAMINATION : OCTOBER 2022

(Examination conducted in December, 2022)

BDA 1321: LINEAR ALGEBRA and LINEAR PROGRAMMING

Duration: 2 Hours

Max. Marks: 50

This question paper contains TWO printed pages and THREE parts.

PART-A

Answer all questions

5x1=5

1. Is the set of vectors $\{(1, 2, 5), (3, 0, 1), (0, 0, 0)\}$ linearly dependent?
2. Is $\{(x, y) : x \geq 0\}$ a subspaces of \mathbb{R}^2 ?
3. What is the dimension of the vector space $\mathbb{M}_{3 \times 3}$ (the set containing all 3 by 3 real matrices) over \mathbb{R} ?
4. Consider the map $T(x, y) = (x + 1, y + 1)$. Is T linear?
5. Find the eigenvalues of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ -1 & 1 & 0 \\ 4 & 6 & 0 \end{bmatrix}$.

PART-B

Answer any 5 questions

5x3=15

6. Compute the length of the vector $(-3, 4, 2)$.
7. Find the angle between the vectors $(4, 3, 1)$ and $(-1, 1, 1)$.
8. Find a unit vector in the direction of $(3, 4, 1)$.
9. Obtain the general vector for \mathbb{R}_{30}^2 .
10. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a map defined by $T(\mathbf{v}) = \|\mathbf{v}\|$. Is T linear? Justify your answer.
11. Find the standard matrix for the dilation transformation $T(\mathbf{x}) = 3\mathbf{x}$ for \mathbf{x} in \mathbb{R}^2 .
12. Define a positive definite matrix. Is the matrix $\begin{bmatrix} 1 & -2 \\ -2 & 6 \end{bmatrix}$ positive definite?

PART-C

Answer any 3 full questions

3x10=30

13. a) Let $u = (3, -1)$ and $v = (-2, 5)$. Sketch the vectors $u, -u, v, 3v$, and $u - 2v$ on a xy - plane. **[5]**
- b) Determine whether the set $\{(1, 2, 3), (4, 5, 6), (2, 1, 0)\}$ is linearly independent or dependent. **[5]**
14. a) Let $v_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ -1 \\ 8 \end{bmatrix}$. Is \mathbf{b} in the $\text{span}(v_1, v_2, v_3)$? **[6]**
- b) Verify Schwarz inequality and Triangle inequality for the vectors $(3, 4)$ and $(4, 3)$. **[4]**
15. a) Define a vector space. **[2]**
- b) Prove that \mathbb{M} , (the set containing all 2 by 2 real matrices) is a vector Space. Also, write a basis for this vector space. **[5]**
- c) Prove that \mathbb{D} , (the set of all diagonal matrices) is a subspace of \mathbb{M} . **[3]**
16. (a) Solve the system of linear equations:

$$4x - 2y + 5z = 6$$

$$3x + 3y + 8z = 4$$

$$x - 5y - 3z = 5$$

[5]

- (b) Find the eigenvalues of $A^2 + A + 4I$, where $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ **[5]**