



Date:

Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27

**B.Sc. (MATHEMATICS) - III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2022**

(Examination conducted in December 2022)

MT 322: MATHEMATICS III

Time- 2hrs

Max Marks-60

This question paper contains **TWO** printed pages and **FIVE** parts.
Scientific calculators are allowed.

I. Answer any SIX of the following questions (6X2=12)

1. Find $\varphi(50)$, where φ is an Euler totient function.
2. Show that $aH = bH$ if and only if $a^{-1}b \in H$ where H is a subgroup of the group G and $a, b \in G$.
3. Find the infimum and supremum of the sequence $\{2^{(-1)^{n+1}}\}$.
4. Find the limit of the sequence $\left\{ \frac{3n^2 - 6n}{5n^2 + 4} \right\}$.
5. Solve $(D^2 + D + 1)y = 0$, where $D = \frac{d}{dx}$.
6. Solve the wronskian of $u = xe^x$ and $v = \sin x$.
7. Find $L[(t-1)^2]$.
8. Find $L[t \sin(at)]$.

II. Answer any TWO of the following questions (2X6=12)

9. Let G be a cyclic group and $a \in G$. If a is a generator of G then prove that a^{-1} is also a generator of G .
10. State and prove Lagrange's theorem for finite groups.
11. Prove that center of a group G defined by $Z(G) = \{a \in G : ax = xa, \forall x \in G\}$ is normal in G .

III. Answer any TWO of the following questions (2X6=12)

12. Prove that the sum of two convergent sequence is convergent.
13. If $x_1 = \sqrt{6}$ and $x_{n+1} = \sqrt{6x_n}$, show that $\{x_n\}$ converges to 6.
14. Discuss the nature of the sequence $\{x^{1/n}\}$ where $x > 0$.

IV. Answer any TWO of the following questions

(2X6=12)

15. Solve $(D^3 - 2D^2 + D)y = e^{2x} + x^2 + x$ where $D = \frac{d}{dx}$.
16. Solve $x \frac{d^2y}{dx^2} - (2x-1) \frac{dy}{dx} + (x-1)y = 0$, $x > 0$, given that e^x is a solution.
17. Solve $y'' + 9y = \sec 3x$ by the method of variation of parameters.

V. Answer any TWO of the following questions

(2X6=12)

18. (a) If $L[f(t)] = F(s)$ then prove that $L[e^{at} f(t)] = F(s-a)$.
(b) Show that $L[e^{kt}] = \frac{1}{s-k}$, $s-k > 0$.
19. Obtain the Laplace transform of $\frac{\sinh t}{t}$.
20. Solve the initial value problem $y' - 5y = e^{2x}$ given $y(0) = 2$.
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