# ST. JOSEPH'S UNIVERSITY, BENGALURU -27 <br> M.Sc. (PHYSICS) - I SEMESTER <br> SEMESTER EXAMINATION: OCTOBER 2022 <br> (Examination conducted in December 2022) PH7321- NUMERICAL TECHNIQUES 

## Time: 2 Hour

Max Marks: 50
This paper contains 2 printed pages and no parts

Answer any 5 questions. Each question carries 10 marks. $5 \times 10=50$

1. a. Using Lagrange interpolation find the slope of the curve at $x=2$ when the curve passes through the points $(0,18),(1,10),(3,-18)$, and $(6,90)$.
b. Using the data given below construct Newton's interpolating polynomial and also find the value of $y$ at $x=5$.

| x | 0 | 10 | 20 | 30 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 7 | 18 | 32 | 48 | 85 |

2. a. A slider in a machine moves along a fixed straight rod. Its distance x cm along the rod is given below for various values of the time $t$ seconds. Find the velocity of the slider and its acceleration when $\mathrm{t}=0.3$ seconds.

| t | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| x | 30.13 | 31.62 | 32.87 | 33.64 | 33.95 | 33.81 | 33.24 |

b. Evaluate the integral $\int_{0}^{1} \frac{x^{2}}{1+x^{3}} d x$ using Simpson's $1 / 3$ rule. Compare the error with the exact value. Take $\mathrm{h}=0.25$.
3. The concentration of salt $x$ in a home-made soap maker is given as a function of time by $\frac{d x}{d t}=37.5-3.5 x$.
At the initial time, $\mathrm{t}=0$, the salt concentration in the tank is $50 \mathrm{~g} / \mathrm{L}$. Using RK-2 Heun's method and a step size of $\mathrm{h}=1.5 \mathrm{~min}$, what is the salt concentration after 3 minutes?
4. a. Find the Fourier transform of $e^{-(a x)^{2}}, a>0$. Hence deduce that $e^{\frac{-x^{2}}{2}}$ is selfreciprocal in respect of the Fourier transform.
b. Find the Fourier transform of $e^{-2(x-3)^{2}}$ using change of scale and shifting property.
5. a. For variable X with the pdf given below, find the mean and variance.

$$
f(x)=\left\{\begin{array}{cc}
\frac{1}{2} x, & 0 \leq x \leq 2  \tag{5}\\
0, & \text { elsewhere }
\end{array}\right.
$$

b. Find the expectation, variance, and standard deviation of the number of Heads in the three-coin toss experiment.
6. The finish times for marathon runners during a race are normally distributed with a mean of 195 minutes and a standard deviation of 25 minutes.
a. What is the probability that a runner will complete the marathon within 3 hours?
b. Calculate to the nearest minute, the time by which the first $8 \%$ of runners have completed the marathon.
c. What proportion of the runners will complete the marathon between 3 hours and 4 hours?
7. a. The rank of 10 students of the same batch in two subjects, A and B are given below. Calculate the rank correlation coefficient.
(5)

| Rank in <br> subject A | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Rank in <br> subject B | 6 | 7 | 5 | 10 | 3 | 9 | 4 | 1 | 8 | 2 |

b. Sam found how many hours of sunshine vs how many ice creams were sold at the shop from Monday to Friday. Fit the best straight line that suits the data given below.

| ' $x$ ' hours of sunshine | 2 | 3 | 5 | 7 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| ' $y$ ' ice creams sold | 4 | 5 | 7 | 10 | 15 |

