Registration Number:

Date & session:

ST. JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. (STATISTICS) – I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2022
(Examination conducted in December 2022)

ST 7121 – PROBABILITY THEORY

Time: 2 Hours **Max Marks: 50**

This paper contains TWO printed pages and ONE part

PART-A

Answer FIVE FULL Questions

1. A) Define monotonic sequence of sets. For a monotonically decreasing sequence of sets prove that $\lim_{n\to\infty} A_n = \cap_{n=1}^{\infty} A_n$.

   B) Define a measure. With usual notations prove that

   $$\mu(A \cup B) = \mu(A) + \mu(B) - \mu(A \cap B)$$

   C) For a continuous random variable X show that $P(X = a) = 0$, a is a real number.

   (4+4+2)

2. A) Prove that every σ field is a field but converse is not true.

   B) With usual notation prove that $P(A^c) = 1 - P(A)$. (8+2)

3. A) Define convergence in probability. With usual notations prove that $X_n \to^r X \Rightarrow X_n \to^P X$.

   B) Define quantile function. Obtain the same for the probability distribution with pdf

   $$f(x) = \begin{cases} kx^{k-1}e^{-x^k}, & x > 0, k > 0 \\ 0 & Otherwise \end{cases}$$

   C) Define convergence in law with an example. (4+4+2)

4. A) Prove that a distribution function F is non decreasing and right continuous. (6)

   B) Obtain the moment generating function of gamma distribution. (4)

5. A) State and prove Holder's inequality. (8)

   B) For a random variable with following probability density function find $E(X)$ (2)

   $$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0 & Otherwise \end{cases}$$

6. A) State and prove Chebycheff inequality.

   B) Prove that characteristic function is uniformly continuous on **R**. (5+5)

7. A) State and prove inversion theorem. (8)

   B) Show that set of natural numbers is Borel set. (2)