## ST. JOSEPH'S UNIVERSITY, BENGALURU -27

M.Sc. (STATISTICS) - I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2022
(Examination conducted in December 2022)
ST 7421 - Mathematical Analysis and Linear Algebra
Time: 2 Hours
Max Marks: 50
This paper contains TWO printed pages and ONE part

## PART-A

## Answer FIVE FULL Questions

1. A) Define (i) Interior point of a set (ii) Limit point of a set with an example each.
B) Define Metric Space. Show that the following are metric spaces. Show that $\mathrm{d}(\mathrm{x}, \mathrm{y})=|\mathrm{x}-\mathrm{y}|$ is a metric for $x, y \in R$.
2. A) Define point-wise Continuity and Uniform Continuity of a function.
B) Prove that any function continuous in closed interval is uniformly continuous. (3+7)
3. A) State and prove Cauchy's general principal of convergence of sequence..
B) Prove that absolute Convergence of a series of arbitrary terms implies its convergence.
4. A) Examine for uniform convergence of $f_{n}(x)=x / n, 0 \leq x \leq \alpha<\infty$.
B) State and Prove Cauchy's criterion for uniform convergence of sequence of functions.
5. A) Define Radius of convergence of power series. Obtain radius of convergence of $\sum_{n=0}^{\infty} x^{n} / n^{2}$.
B) if $\int_{a}^{b} f_{1}(x) d g(x)$ and $\int_{a}^{b} f_{2}(x) d g(x)$ exists then prove that

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\begin{equation*}
\int_{a}^{b}\left(f_{1}(x)+f_{2}(x)\right) d g(x)=\int_{a}^{b} f_{1}(x) d g(x)+\int_{a}^{b} f_{2}(x) d g(x) \tag{4+6}
\end{equation*}
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6. A) Evaluate the double integral $\iint_{S}\left(2 x^{3}+y^{3}\right) d(x, y), S=\left\{(x, y): x \geq 0, x^{2}+y^{2} \leq 1\right\}$.
B) State and prove completion theorem.
7. A) Define a basis of a vector space and subspace of a vector space and dimension of a vector space.
B) Find row echelon form of the matrix $A=\left[\begin{array}{lll}1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 6 & 9\end{array}\right]$. Hence find the rank $A$.
c) Define quadratic forms and different classifications of quadratic forms.
