Registration Number:

Date & session:

ST. JOSEPH'S UNIVERSITY, BENGALURU -27

M.Sc. (STATISTICS) – III SEMESTER

SEMESTER EXAMINATION: OCTOBER 2022

(Examination conducted in December 2022)

ST 9120 – STOCHASTIC PROCESSES

Time: 2 ½ Hours **Max Marks: 70**

This paper contains TWO printed pages and TWO parts

PART-A

Answer any SIX of the following **6x 3= 18**

1. Define Markov chain with an example.
2. Define the irreducible Markov chain and construct an example for the same.
3. Define following terms

   (i) transient state (ii) Random walk (iii) Stochastic matrix
4. What do you mean by period of a state? Find the period of all the states for a Markov chain $\{X_n\}$ with state space S= {1,2,3} and TPM, $P = \begin{bmatrix} 0 & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$.
5. Define independent and stationary increments of a stochastic process.
6. Distinguish between counting process and Poisson process with an example.
7. Explain renewal process with an example.
8. Calculate the probability of ultimate extinction of a Branching process whose probability generating function of the offspring distribution is given by

   $\emptyset(s) = \frac{1}{5} + \frac{s}{5} + \frac{3s^2}{5}$.

PART B

Answer any FOUR of the following **4 x 13 = 52**

9. A) For a homogeneous Markov chain $\{X_n\}$ with state space S= {1,2,3} and initial probabilities as 1/6, 1/3 and 1/2 and TPM $\mathrm{P} = \begin{bmatrix} 0 & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$,

   Find $(i) P(X_3 = 1/X_1 = 2)$ $(ii) P(X_5 = 3/X_2 = 2)$ $(iii) P(X_1 = 2)$

   B) Obtain the stationary probabilities for the Markov chain in 9 A).

   C) Define first passage probability with an example. (6+5+2)

10. A) Show that $\{Y_n\}$ is a Markov chain, where $Y_n = \max(X_1, X_2, \dots, X_n), X_n \sim B(2, p)$ for $n = 1,2, \dots,$ and $X_i's$ are i.id. Obtain the transition probability matrix of $\{Y_n\}$

    B) Prove that

    (i) if state i is recurrent and $i \leftrightarrow j$ then j is also recurrent.

    (ii) if state i is transient and $i \leftrightarrow j$ then j is also transient. (6+7)

11. A) Explain the gambler's ruin problem. Obtain the probability of a ruin of a player.

    B) Define mean recurrence time. If $f_{jj}^{(1)} = \frac{1}{2}$ & $f_{jj}^{(n)} = \left(\frac{1}{3}\right)^n, n \geq 2$ obtain the mean recurrence time for state j. (9+4)

12. A) Stating the postulates of the Poisson process prove that $P(N(t) = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$.

    B) Prove the Chapman-Kolmogorov equation for continuous time Markov chain.

    (10+3)

13. A) For a Poisson process $\{N(t)\}$ prove that correlation between $N(t+s)$ and $N(t)$ is $\sqrt{\frac{t}{t+s}}$

B) Derive the renewal equation.

C) Define renewal function. With usual notation prove that $m(t) = \sum_{n=1}^{\infty} F_n(t)$. (4+5+4)

14. A) For a Galton-Watson branching process $\{X_n\}$ with $X_0 = 1$, mean and variance of offspring distribution as m and σ^2 respectively show that

$$E(X_n) = m^n \text{ and } V(X_n) = \begin{cases} \frac{m^{n-1}(m^n-1)\sigma^2}{(m-1)} & when\ m \neq 1 \\ n\sigma^2 & when\ m = 1 \end{cases}$$

B) Define martingale. For $X_n = \prod_{i=1}^{n} Z_i$, where $Z_i's$ are independent random variables with mean 1 show that show that $\{X_n\}$ is a Martingale. (9+4)