ST JOSEPH'S UNIVERSITY, BENGALURU-27
B.Sc. – II SEMESTER
SEMESTER EXAMINATION: APRIL 2023
(Examination conducted in May 2023)
MT 221: MATHEMATICS-II
(For current batch students only)

Time: 2 Hours   **Max Marks: 60**
This paper contains TWO printed pages and THREE parts.

PART-A
Answer any SIX of the following: **(2 X 6 =12)**

1. Define order of an element of a group. Also find the order of the element i of the multiplicative group $G = \{1, -1, i, -i\}$.

2. Evaluate $\int_0^\pi x \sin^3 x \ dx$.

3. Find the area of the Lemniscate $r^2 = a^2 \cos 2\theta$

4. Show that the pedal equation of the cardioid $r = a(1 - cos\theta)$ is $2ap^2 = r^3$.

5. For the curve $2s = y^2$, show that $\frac{dy}{dx} = \frac{1}{\sqrt{y^2-1}}$

6. Find the integrating factor of $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$

7. Test the exactness and solve $(e^y + 1) \cos x \ dx + e^y \ sinx \ dy = 0$.

8. Obtain the orthogonal trajectories of the family of circles $r = c \cos \theta$, where c is a parameter.

PART-B

Answer any Three of the following: **(3X6=18)**

9. Prove that $G = \{2,4,6,8\}$ forms an abelian group under $\times_{10}$.

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10. Prove that a non-empty subset H of a group $(G,*)$ is a subgroup of G if and only if for every $a, b \in H, a * b^{-1} \in H$.

11. Evaluate (i) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^8 x \, dx$ (ii) $\int_0^1 x^{\frac{3}{2}} (1-x)^{\frac{3}{2}} dx$. (2+4)

12. Find the surface area of the solid generated when the cardioid $r = a(1 + \cos\theta)$ revolves about the initial line.

<center>PART-C</center>

Answer any Five of the following: (5X6=30)

13. Derive the formula for the angle between the radius vector and the tangent at any point on a curve $r = f(\theta)$.

14. Show that the radius of curvature at any point on the curve $r = a(1 - \cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$.

15. Find all the asymptotes of the curve $x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0$

16. (i) Find the envelope of the family of curves $(x - c)^2 + y^2 = 4c$, where c is the parameter.
    (ii) Solve: $\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1} x}}{1+x^2}$ (3+3)

17. Solve: $\frac{dy}{dx} - \frac{y}{x} = 2e^{-x}y^2$

18. By finding the suitable integrating factor solve: $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$

19. Show that the family of parabolas $y^2 = 4a(x + a)$ is self-orthogonal.

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