ST. JOSEPH'S COLLEGE(AUTONOMOUS), BENGALURU -27
B.Sc (MATHEMATICS) - IV SEMESTER
SEMESTER EXAMINATION: APRIL 2023
(Examination conducted in May 2023)
MT 422- MATHEMATICS IV

(For current batch students only)

Time: 2 Hours   Max Marks: 60

This paper contains **TWO** printed pages and **THREE** parts.

PART A

Answer any SIX of the following.   **[6X 2=12]**

1. Determine whether $\phi : (\mathbb{R}, +) \to (\mathbb{R}^*, \times)$ defined by $\phi(x) = e^x$ is a group homomorphism or not.

2. Examine the convergence of the series $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{4} + \ldots\ldots$

3. State D'Alembert's ratio test.

4. Determine whether the function $f(x) = \left(\dfrac{\pi - x}{2} \right)^2$ in $0 < x < 2\pi$ is even or an odd function.

5. Define gamma function. Find $\gamma(5)$

6. Find $\beta(1, 6)$

7. Given the differential equation $\dfrac{dy}{dx} = x + y$ with $y(0) = 1$. Find $y(0.1)$ using Euler's method .

8. Construct the finite difference table for the following data

x	45	50	55	60	65
f(x)	0	3	14	69	228

PART B

Answer any THREE of the following.   **[3X 6=18]**

9. State and prove the First Isomorphism theorem for groups.

10. Examine the convergence of the series $\dfrac{1}{\sqrt{1}+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \dots..$

11. State and prove Cauchy's root test.

12. Find the root using Secant method for $f(x) = x^3 - 4x - 9$ in the interval $(2, 3)$ upto three decimal places.

PART C

Answer any FIVE of the following.   **[5X 6=30]**

13. Obtain the Fourier series expansion for the function $f(x) = x^2$ over $(-\pi, \pi)$

14. Obtain the half range Fourier sine series for the function $f(x) = e^{-ax}$ over $(0, 1)$.

15. Prove that $\beta(m, n) = \dfrac{\gamma(m)\gamma(n)}{\gamma(m+n)}$

16. (a) Prove that $\gamma(n+1) = n\gamma(n)$

    (b) Using the trapezoidal rule, evaluate the integral $\displaystyle\int_0^2 e^{x^2} dx$ by taking the step size $h = 0.5$ **[3+3]**

17. Find the value of $y(1925)$ from the following data

x	1891	1901	1911	1921	1931
f(x)	46	66	81	93	101

18. Determine the value of $y(0.1)$ for the differential equation $\dfrac{dy}{dx} = y - x$ given that $y(0) = 2$ using the Runge-Kutta fourth order method by taking $h = 0.1$.

19. Evaluate $\displaystyle\int_0^1 \dfrac{dx}{x^3 + x + 1}$ using Simpson's 3/8th rule by taking the step size $h = 1/6$.

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