Registration Number:
Date & Session

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU -27

B.Sc (MATHEMATICS) - VI SEMESTER

SEMESTER EXAMINATION: APRIL 2023

(Examination conducted in May 2023)

MT6218-MATHEMATICS VIII

(For current batch students only)

Time: 2 ½ Hours **Max Marks: 70**

This paper contains TWO printed pages and THREE parts.

PART-A

Answer any FIVE of the following questions. **(5X2=10)**

1. Evaluate $\int_c (x\,dy - y\,dx)$ along the curve $y = x^2$ from origin to $(2,2)$.
2. Evaluate $\int_0^b \int_0^a (x^2 + y^2)\,dx\,dy$.
3. Evaluate $\int_0^1 \int_0^2 \int_0^3 (xyz)\,dz\,dy\,dx$.
4. State Green's theorem in the plane.
5. Using Gauss divergence theorem show that $\iint_S r.n\,ds = 3V$ where V is the volume of the space bounded by the surface S and $r^2 = x^2 + y^2 + z^2$.
6. Find the Laplace transform of $(1+t)^2$.
7. If $L[f(t)] = F(s)$ then show that Laplace transform of $e^{at}[f(t)] = F(s-a)$.
8. State convolution theorem.

PART-B

Answer any SEVEN of the following questions. **(7X6=42)**

9. Show that $\int_c (2xy)\,dx + (x^2 + 2yz)\,dy + (y^2 + 1)\,dz$ is independent of path along the curve c leading from the origin to the point $(1,1,1)$ and hence evaluate.
10. Evaluate $\iint_R x^2 y^2\,dx\,dy$ where R is the triangular region with vertices $(0,0)$, $(2,0)$ and $(2,3)$.

11. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$ by changing the order of integration.

12. Evaluate $\iint xy\,dx\,dy$ over the positive quadrant bounded by the circle $x^2 + y^2 = 1$.

13. Find the surface area of the sphere $x^2 + y^2 + z^2 = 4$ using double integration.

14. Evaluate $\iiint_R \frac{dx\,dy\,dz}{(x+y+z)^3}$ where R is the region bounded by the coordinate planes and the plane $x + y + z = 1$.

15. Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2 dy$ where c is the closed curve bounded by $y = x$ and $y = x^2$.

16. State Gauss divergence theorem and evaluate using Gauss divergence theorem for $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ over the rectangular parallelepiped bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 2, z = 3$.

17. State and prove Stoke's theorem.

PART-C

<u>Answer any TWO of the following questions.</u> **(3X6=18)**

18. Find the Laplace transform of the function $t^2 \cos at$.

19. A periodic function of period $\frac{2\pi}{\omega}$ is defined by $f(t) = \begin{cases} \sin \omega t, & 0 \le t \le \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega} \end{cases}$,

    where ω is a constant. Show that $L\{f(t)\} = \frac{\omega}{(s^2 + \omega^2)\left(1 - e^{\frac{-s\pi}{\omega}}\right)}$.

20. Find the inverse Laplace transform of the function $\frac{s}{s^2 + s - 2}$.

21. Verify convolution theorem for the functions $f(t) = t$ and $g(t) = e^t$.

22. Using Laplace transform method solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}$, given that $y(0) = 0, y'(0) = 0$.