Register Number:

Date and Session:

ST. JOSEPH'S COLLEGE(AUTONOMOUS), BENGALURU -27
B.Sc (MATHEMATICS) - V SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November/December 2023)
MT 5223- MATHEMATICS VI

(For current batch students only)

Time: 2 Hours | Max Marks: 60

This paper contains **TWO** printed pages and **THREE** parts.

PART A

Answer any SIX of the following. **[6X 2=12]**

1. Find the locus of z such that $im(z+i) \geq 0$
2. Show that $u = e^x cos(y)$ and $v = e^x sin(y)$ are orthogonal to each other.
3. Check if the function $v = 2xy$ is harmonic.
4. Explain the inversion of a complex function.
5. Find the fixed points of $w = \dfrac{i-z}{z+i}$.
6. If $\phi = x^2y^2z^2$, find $\nabla\phi$.
7. If $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, then show that $curl\vec{F} = 0$.
8. Find the spherical co-ordinates of the cartesian points $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$.

PART B

Answer any FIVE of the following. **[5X 6=30]**

9. State and prove the necessary condition for a complex function $f(z) = u + iv$ to be analytic.
10. If $f(z) = u + iv$ is analytic then show that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$.

11. Construct an analytical function $f(z) = u + iv$ whose $u - v = x^3 + 3x^2y - 3xy^2 - y^3$.

12. Find the bi-linear transformation which maps $1, -i, -1$ to $0, i, \infty$. Also, find its invariant points.

13. If $f(z)$ is analytic within and on a simple closed curve C and if $'a'$ is any point within C then prove that $f(a) = \frac{1}{2\pi i} \oint\limits_C \left(\frac{f(z)}{z-a}\right) dz$.

14. Evaluate $\oint\limits_C \frac{sin(\pi z^2) + cos(\pi z^2)}{(z-1)(z-2)}\, dz$ where $C : |z| = 3$.

15. State and prove Lioville's theorem.

PART C

Answer any THREE of the following. **[3X 6=18]**

16. Find the unit vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$.

17. Find the angle between the directions of the normals to the surface $xy = z^2$ at the points $(4, 1, 2)$ and (3,3,-3).

18. Define Laplacian of a scalar point function. Prove that $\nabla^2(\phi\psi) = \phi\nabla^2\psi + 2\nabla\phi\nabla\psi + \psi\nabla^2\phi$.

19. Show that $\vec{F} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is a conservative force field and find its scalar potential.

20. Prove that spherical coordinate system is an orthogonal curvilinear coordinate system.

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