Register number:

Date and session:

ST. JOSEPH'S UNIVERSITY, BENGALURU- 27
M.Sc MATHEMATICS- I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November/December)
MT 7421- Ordinary Differential Equations

Duration: 2 Hours **Max. Marks:** 50

1. The paper contains **ONE** printed pages.
2. Answer any **FIVE FULL** questions, where each question carries 10 marks.

1. State and prove Abel's Formula.
2. (a) Show that the solution set of the equation $y'' + 7y' + 12y = 0$ forms the fundamental set.
   (b) A tank initially contains 50 gallons of pure water. A salt solution containing 2 pounds of salt per gallon of water is poured into the tank at the rate of 3 gallons per minute. The mixture is stirred and is drained out of the tank at the same rate.
   i. Find the initial value problem that describes the amount of salt in the tank at any time.
   ii. Find the amount of salt in the tank at any time.
   iii. Find the amount of salt in the tank after 20 minutes.
   iv. Find the amount of salt in the tank after a long time. **[3+7]**
3. Find the power series solution of the differential equation $(x^2 + 1)y'' + xy' - xy = 0$ in powers of x .
4. Solve using Frobenius method the given differential equation $xy'' + y' - xy = 0$.
5. Find the eigenvalue and eigen function of the differential equation $y''+\lambda y = 0$ with boundary conditions $y(0) = 0$ and $y(1) = 0$.
6. Find the general solution of $x^2y'' + 7xy' + 8y = 0$ by finding the solution of its adjoint equation.
7. (a) Define the critical point for an autonomous system of differential equations. Find the critical points of $\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{q}{a}sinx = 0$
   (b) Solve the equation $y'' = -2t(y')^2$ with $y(0) = 2, y'(0) = -1$. **[4+6]**

OR

   (a) Determine the type and stability of the critical point of $(0, 0)$ of the non linear system of equation $\frac{dx}{dt} = 8x - y^2$, $\frac{dy}{dt} = -6y + 6x^2$.
   (b) Solve the differential equation $yy'' = (y')^2$. **[4+6]**

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