ST JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. MATHEMATICS - III SEMESTER

SEMESTER EXAMINATION: OCTOBER 2023

Registration Number:
Date \& Session

## MTDE9422: OPTIMIZATION TECHNIQUES

(For current batch students only)
Time: 2 Hours

Max Marks: 50
This paper contains TWO printed pages
Each question carries 10 marks
Answer any FIVE full questions

## Note: TWO graph sheets are to be provided.

1. 

Use simplex method to solve the LPP: Maximize $Z=x+2 y$ subject to the constraints $-x+2 y \leq 8$, $x+2 y \leq 12, x-2 y \leq 3$ and $x, y \geq 0$.
2. (i) Use graphical method to solve the following LPP: Max $Z=x_{1}+x_{2}$ subject to the constraints $x_{1}+$ $\mathrm{x}_{2} \leq 1, \mathrm{x}_{1}+\mathrm{x}_{2} \geq 3$ and $\mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$.
(ii) The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation costs per truckload on the different routes are summarized in the transportation model given below. Find the minimum-cost shipping by North-West Corner method.

|  | D | E | F | G | Availability |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A | 11 | 13 | 17 | 14 | 250 |
| B | 16 | 18 | 14 | 10 | 300 |
| C | 21 | 24 | 13 | 10 | 400 |
| Requirement | 200 | 225 | 275 | 250 |  |

3. A (i) Solve the assignment problem to minimize the total work hour.

|  |  | Men |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 11 | III | , |
|  | A | 8 | 26 | 17 | 11 |
| Jobs | B | 13 | 28 | 4 | 26 |
| Jobs | C | 38 | 19 | 18 | 15 |
|  |  | 19 | 26 | 24 |  |

$A(i i)$ Suppose that there are five jobs, each of which has to be processed on two machines $A$ and $B$ in the order $A B$. Determine a sequence in which these jobs should be processed so as to minimize the total processing time, where the processing times are given in the following table.

| Jobs | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Machine A | 6 | 2 | 10 | 4 | 11 |
| Machine B | 3 | 7 | 8 | 9 | 5 |

(OR)
$B(i) \quad$ Solve the following game using dominance property, whose payoff matrix is as follows.

|  | B1 | B2 | B3 |
| :---: | :---: | :---: | :---: |
| A1 | -5 | 10 | 20 |
| A2 | 5 | -10 | -10 |
| A3 | 5 | -20 | -20 |

B(ii) Explain Minimax regret criterion and Expected opportunity loss criterion.
4. Solve the game with the given payoff matrix by graphical method.
A1
A2
A3
A4
A4
A5 $\left[\begin{array}{cc}1 & 2 \\ 5 & 4 \\ -7 & 9 \\ -4 & -3 \\ 2 & 1\end{array}\right]$
5. (i) Explain Floyd's algorithm
(ii) Determine the value for the game and check if it is Strictly Determinable.

|  | B1 | B2 | B3 |
| :---: | :---: | :---: | :---: |
| A1 | 1 | 3 | 1 |
| A2 | 0 | -4 | -3 |
| A3 | 1 | 5 | -1 |

6. Determine the early start time and late finish time in respect of all node points and identify critical path for the following network.

| Activity | $1-2$ | $1-3$ | $1-4$ | $2-5$ | $4-6$ | $3-7$ | $5-7$ | $6-7$ | $5-8$ | $6-9$ | $7-10$ | $8-10$ | $9-10$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Duration | 10 | 8 | 9 | 8 | 7 | 16 | 7 | 7 | 6 | 5 | 12 | 13 | 15 |

7. (i) Find the time estimate $\left(\mathrm{t}_{\mathrm{e}}\right)$ for the following project and construct the Network Diagram.

| Activity | $1-2$ | $1-3$ | $1-4$ | $2-4$ | $3-4$ | $3-5$ | $4-6$ | $4-7$ | $5-7$ | $6-7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Optimistic time $\left(\mathrm{t}_{0}\right)$ | 4 | 2 | 6 | 1 | 6 | 6 | 3 | 4 | 2 | 2 |
| Most Likely time $\left(\mathrm{t}_{\mathrm{m}}\right)$ | 6 | 3 | 8 | 2 | 7 | 7 | 5 | 11 | 4 | 9 |
| Pessimistic time $\left(\mathrm{t}_{\mathrm{p}}\right)$ | 8 | 10 | 16 | 3 | 8 | 14 | 7 | 12 | 6 | 10 |

(ii) Customers arrive at one-man barber shop according to a Poisson process with a mean inter arrival of 12 min . Customers spend an average of 10 min in the barber's chair.
(a) Calculate the percentage of customers who have to wait prior to getting into the barber's chair.
(b) What is the probability that more than 3 customers are in the system?
(c) What is the average time customer spend in the queue?

