Registration Number:

Date & session:

ST.JOSEPH'S UNIVERSITY, BENGALURU -27

M.Sc (PHYSICS) – I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2023

(Examination conducted in November/December 2023)

PH 7123 – CLASSICAL MECHANICS

(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains 2 printed pages and 2 parts

PART A

Answer any FIVE full questions. **(5x7=35)**

1. With the transformation equations for spherical polar coordinate to the Cartesian system being given by: $x=r\ \sin\theta\cos\phi,\ \ y=r\ \sin\theta\sin\phi,\ \ z=r\ \cos\theta$, obtain the transformation matrix for the basis vectors: $\begin{pmatrix}\hat{\boldsymbol{i}}\\ \hat{\boldsymbol{j}}\\ \hat{\boldsymbol{k}}\end{pmatrix}$ and $\begin{pmatrix}\hat{\boldsymbol{r}}\\ \hat{\boldsymbol{\theta}}\\ \hat{\boldsymbol{\phi}}\end{pmatrix}$.

2.
   (a) Using the Hook's Law: $F=-kx$ write down the equation of motion for a simple harmonic oscillator of mass m .
   (b) Write down the general solution for this equation of motion.
   (c) Using the initial condition that at $t=0$, $x=0$ and $v_x=v_0$ obtain the expression for the displacement as a function of time.
   (d) From your result in (c), obtain the expression for the momentum of the particle.
   (e) Obtain the expression for the momentum in terms of the displacement. [1+1+1+1+3]

3.
   (a) Write down the Lagrange Equation in terms of the generalized coordinate q_k .
   (b) From this, write down the generalized momentum of the system.
   (c) What is a cyclic coordinate?
   (d) With a neat diagram show that a symmetry to linear translation of the system, results in the conservation of linear momentum of the system. [1+1+1+4]

4.
   (a) What is Hamilton's Principle (state it mathematically; describe all quantities and how they are related to a system)?
   (b) Find the extremum of the following functional: $J=\int_b^a \left[3x+\sqrt{\frac{\partial}{\partial x}y(x)}\right]dx$ [2+5]

5.

(a) Write down the Lagrangian of the equivalent one body problem for a Central Force Field described by a potential $V(r)$.

(b) How many degrees of freedom does the reduced mass have in this potential?

(c) Compute the equation of motion for each degree of freedom.

(d) Use Routh's Procedure to eliminate the term from the cyclic coordinate in the equation of motion. [2+1+3+1]

6. The total energy of the reduced mass in a central force field obeying the attractive inverse square force law is given as: $E=\frac{1}{2}m\dot{r}^2+\frac{\ell^2}{2mr^2}-\frac{k}{r}$

(a) For $E>0$

i. obtain the expressions for distances of closest approach.

ii. are all these expressions valid?

iii. What kind of orbits does this condition allow?

(b) For $E<0$

i. Obtain the expressions for distances of closes approach.

ii. What do these distances signify? [2+1+1+2+1]

7.

(a) Explain Legendre Transformation (mathematically clearly indicating all the equations involved).

(b) Using the Legendre Transformation, explain how we can obtain the Hamiltonian from the Lagrangian (clearly mention what variables the Lagrangian and Hamiltonian are dependent on and how your definition of Legendre Transformation differs from how the Hamiltonian is obtained from the Lagrangian).

(c) From the above, obtain the Hamilton Equations. [3+2+2]

PART-B

Answer any THREE full questions **(3x5=15)**

8. Show that the virtual work due to constraint forces vanishes for the Atwood Machine.

9.

(a) Write down the Lagrangian for an object of mass m falling vertically under gravity.

(b) From the Lagrangian, obtain the equation of motion of the particle. [3+2]

10. The distance between the Sun and Earth is $r=1.4861\times10^{11}\ \mathrm{m}$ and their masses are respectively given by $M_{\mathrm{Sun}}=1.9891\times10^{30}\ \mathrm{kg}$ and $M_{\mathrm{Earth}}=5.972\times10^{24}\ \mathrm{kg}$. Obtain the distance of the Center of Mass of the Sun-Earth system in units of Radius of Sun $R_{\mathrm{Sun}}=6.9634\times10^{8}\ \mathrm{m}$.

11. The Lagrangian of a system is given as: $L=\frac{1}{2}m\ell^2\dot{\theta}^2+mg\ell\cos\theta$ where m and ℓ are constants and θ is the coordinate attached to the degree of freedom. Compute the Hamiltonian of the system.