Registration Number:
Date & Session:

ST JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. PHYSICS – III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November /December 2023)
PH 9120: QUANTUM MECHANICS II
(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains 4 printed pages and 2 parts

PART A

Answer any FIVE full questions. **(5x7=35)**

1. What is parity operator? Show that if potential energy function V(x)=V(-x) then parity operator commutes with Hamiltonian

2. a) Derive an expression for the first order perturbation to the energy for the non-degenerate case

   b) The momentum operator is given $\hat{P}=-i\hbar\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)$ find the expectation value of $\hat{P}$ in the ground state of the hydrogen atom (The wave function for the ground state of Hydrogen atom is: $\psi_{100}(r,\theta,\phi)=\frac{1}{\sqrt{\pi a^3}}e^{-r/a}$ (5+2)

3. a)The Hydrogen Atom wave function is given as:

$$\psi_{nlm}(r,\theta,\phi)=\sqrt{\left(\frac{2}{na}\right)^3\frac{(n-l-1)!}{2n[(n+l)!]^3}}\,e^{-r/na}\left(\frac{2r}{na}\right)^l L_{n-l-1}^{2l+1}\left(\frac{2r}{na}\right)Y_l^m(\theta,\phi)$$

   where $Y_l^m(\theta,\phi)=(-1)^{|m|}\sqrt{\frac{2l+1}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}\,P_l^m(\cos\theta)e^{im\phi}$ are the spherical harmonics compute Y_{00} and Y_{10}

   b) Find the ground state energy of an anisotropic harmonic oscillator described by the potential $V(x,y,z)=\frac{1}{2}m\omega^2x^2+2m\omega^2y^2+8m\omega^2x^2$ in units $\hbar\omega$ (4+3)

4. a) Write the Schrodinger equation for the hydrogen atom using Spherical Polar coordinate system

b) The problem of determining the differential cross section($\frac{d\sigma}{d\Omega}$) always reduces to that of obtaining scattering amplitude $f(\theta,\phi)$. Justify. (3+4)

5. Consider a one dimensional potential well without any rigid walls, using WKB approximation to obtain the quantization condition of energy levels of bound states.

6. Show that the transition probability per unit time for a system to make a transition from initial state 'm' to final state 'k', where m- is a discrete state and k- is one among the set of final density of states is given by $w=\frac{2\pi}{\hbar}|\langle k_j|H'|m\rangle|^2\rho(E_j)$ Where, the symbols have usual meaning.

7. Calculate the scattering amplitude for a spherical symmetry potential in the first order Born approximation and discuss its condition of validity.

PART-B

Answer any 3 questions
(3x5=15)

8. If the perturbation H'=ax where a is a constant, is added to the infinite square well potential $V(x)=\begin{cases}0 & 0\leq x\geq a\\ \infty & Otherwise\end{cases}$ Find the first order correction to the ground state energy. $\psi_n=\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$

9. Consider a system of two non-interacting identical fermions, each of mass m in an infinite square well potential of width a . (Take the potential inside the well to be zero and ignore spin). Find the composite wave function for the system with total energy $\frac{5\pi^2\hbar^2}{2ma^2}$. Usual wave function for the particle in a box is $\psi_n=\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$

10. A particle, initially (i.e t→-∞) in its ground state in an infinite potential well whose walls are located at x=0 and x=a, is subject at time t=0 to a time-dependent

perturbation $v(t)=\varepsilon\hat{x}e^{-t^2}$. Where ε is a small real number. Calculate the probability that the particle will be found in its first excited state after sufficiently long time(i.e t→+∞). (use: $\int_0^a x\sin(\frac{2\pi x}{a})\sin(\frac{\pi x}{a})dx=-\frac{8a^2}{9\pi^2}$)

11. Calculate the differential scattering cross-section in the first order Born approximation for a central potential $V(r)=Z_1 Z_2 e^2/r$. Where, $Z_1 e$ and $Z_2 e$ are the charges of the projectile and target particles respectively.

[Constants: **h**=6.626070x10^{-34} J s (**Planck's constant**), **1eV** = 1.6x10^{-19} J (**electron volt to Joules**), **c**=2.99792458x10^{8} m/s (**speed of light**), **1Å** = 1x10^{-10}m (**Angstrom to meters**), **e** = 1.602176x10^{-19} C (**electronic charge**), ε_0= 8.85418782x10^{-12}m^{-3}kg^{-1}s^{4}A^2 (**permittivity of free space**), m_{proton}=1.672621898x10^{-27}kg (**mass of proton**), $m_{electron}$=9.10938356x10^{-31}kg (**mass of electron**), $m_{neutron}$=1.674927471x10^{-27}kg (**mass of neutron**), **a** = 5.029x10^{-10}m (**Bohr radius**), **α** = 1/137 (**Fine Structure Constant**), **G**=6.674x10^{-11}m^{3}kg^{-1}s^{-2} (**Gravitational constant**), $M_\odot$=1.9891x10^{30} kg (**Solar mass**), $R_\odot$=6.9x10^{8} m (**Sun's Radius**), **σ** = 5.67x10^{-8}

Table of (some) **Integrals**

Gamma Function:
$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$
$\Gamma(n)=(n-1)!$
$\Gamma(\frac{1}{2}+n)=\frac{(2n)!}{4^n n!}\sqrt{\pi}$

(a) $\int_0^\infty e^{-2bt}dt=\frac{1}{2b}$

(b) $\int_0^\infty te^{-2bt}dt=\frac{1}{4b^2}$

(c) $\int_0^\infty t^2e^{-2bt}dt=\frac{1}{4b^3}$

(d) $\int_0^\infty t^3e^{-2bt}dt=\frac{3}{8b^4}$

(e) $\int_0^\infty t^4e^{-2bt}dt=\frac{3}{4b^5}$

(f) $\int_0^\infty t^5e^{-2bt}dt=\frac{15}{8b^6}$

(l) $\int\frac{t^2}{(t^2+b^2)^2}dt=\left(-\frac{t}{(2b^2+2t^2)}+\frac{1}{2b}\tan^{-1}\left(\frac{t}{b}\right)\right)$

(m) $\int\frac{1}{(t^2+b^2)^3}dt=\frac{3}{8b^5}\left(\frac{5/3b^3t+bt^3}{(b^2+t^2)^2}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(n) $\int\frac{t^2}{(t^2+b^2)^4}dt=\frac{1}{16b^5}\left(\frac{bt^5+8/3b^3t^3-b^5t}{(b^2+t^2)^3}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(o) $\int\frac{t^4}{(t^2+b^2)^4}dt=\frac{1}{16b^3}\left(\frac{bt^5+8/3b^3t^3-b^5t}{(b^2+t^2)^3}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(p) $\int\frac{t^6}{(t^2+b^2)^4}dt=\frac{1}{16b}\left(\frac{11bt^5+40/3b^3t^3-5b^5t}{(b^2+t^2)^3}+5\tan^{-1}\left(\frac{t}{b}\right)\right)$

(q) $\int\sqrt{a/x-1}\,dx=x\sqrt{a/x-1}+a\ \tan^{-1}(\sqrt{a/x-1})$

(g) $\int_0^\infty t^6 e^{-2bt} dt = \frac{45}{8b^7}$

(h) $\int \frac{1}{t^2+b^2} dt = \frac{1}{b} \tan^{-1}\left(\frac{t}{b}\right)$

(i) $\int \frac{1}{(t^2+b^2)^2} dt = \frac{1}{2b^3}\left(\frac{bt}{(b^2+t^2)} + \tan^{-1}\left(\frac{t}{b}\right)\right)$

(j) $\int_0^\infty t^4 e^{-\alpha^2 t^2} dt = \frac{3\sqrt{\pi}}{8\alpha^5}$

(k) $\int \frac{1}{(t^2+b^2)^4} dt = \frac{1}{16b^7}\left(\frac{15t^5 b + 40 b^3 5^3 + 33 b^5 t}{(3t^6 + 9bt^4 + 9b^3 t^2 + b^5)} + 5\tan^{-1}\left(\frac{t}{b}\right)\right)$

(r) $\int \sqrt{1-ax}\, dx = -\frac{2(1-ax)^{3/2}}{3a}$

(s) $\int \sqrt{1-ax^2}\, dx = \frac{1}{2} x \sqrt{1-ax^2} + \frac{\sin^{-1}\sqrt{a}\, x}{2\sqrt{a}}$

(t) $\int_{-\infty}^{\infty} e^{-\alpha t^2 + i\omega t} dt = \sqrt{\frac{\pi}{\alpha}}\, e^{-\frac{\omega^2}{4\alpha}}$

(u) $\int_0^\infty t^n e^{-st} dt = \frac{n!}{s^{n+1}}$ (Laplace Transform)