Register Number:
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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - I SEMESTER

SEMESTER EXAMINATION- OCTOBER 2019

PH 7118 - CLASSICAL MECHANICS

Time-2 1/2 hrs. **Maximum Marks-70**

This question paper has 3 printed pages and 2 parts

PART A

Answer any FIVE full questions. **(5x10=50)**

1. The Kinetic Energy of a system made up of N particles can be described as $T=T(q_k,\dot{q}_k,t)=\frac{1}{2}\sum_{i=1}^{N} m_i \dot{\vec{\mathbf{r}}}_i \cdot \dot{\vec{\mathbf{r}}}_i$, where q_k and $\dot{q}_k$ are the generalized coordinates and velocities respectively. The D'Alembert Principle is given as $Q_k=\sum_{i=1}^{N} m_i \ddot{\vec{\mathbf{r}}}_i \cdot \frac{\partial \vec{\mathbf{r}}_i}{\partial q_k}$. Obtain the relationship between Q_k and T .
2. State the Hamilton Principle and obtain the Euler-Lagrange equation from Hamiltion principle for monogenic systems.
3. A particle of reduced mass μ is moving in a central force described by an inverse square law: $F(r)=-\frac{k}{r^2}$, with k being a negative constant (repulsive force). Obtain the expression for the radius of closest approach to the central object if the total energy of the particle is positive (greater than zero).
4. If the Lagrangian of a system is not explicitly dependent on a generalized coordinate show that
   (a) The generalized momentum is a conserved quantity. (**5 Marks**)
   (b) The Hamiltonian is a conserved quantity if and only if the Lagrangian is not explicitly dependent on time. (**5 Marks**)
5. For rotational transformations, we had obtained the transformation matrix for the frames as $R^f(dq_k)=\begin{pmatrix} \cos(dq_k) & \sin(dq_k) \\ -\sin(dq_k) & \cos(dq_k) \end{pmatrix}$ and for vectors as

$$R^v(dq_k) = \begin{pmatrix} \cos(dq_k) & -\sin(dq_k) \\ \sin(dq_k) & \cos(dq_k) \end{pmatrix}$$. Show that:

(a) $R^f(dq_k) = \left(R^v(dq_k)\right)^{-1}$ **(5 Marks)**

(b) $R^f(dq_k + dq_i) = R^f(dq_k)\, R^f(dq_i)$ **(5 Marks)**

6. An infinite elastic rod can be assumed to be described by a series of mass points connected to each other by springs. Obtain the Lagrangian Density $\mathscr{L}$ of such a system and from the Lagrangian density, obtain the Hamiltonian Density: $\mathscr{H} = \Pi\, \dot{\eta} - \mathscr{L}$.
7. From the equations of energy conservation and angular momentum conservation for a particle moving in a central force field, obtain the second integrals of motion.

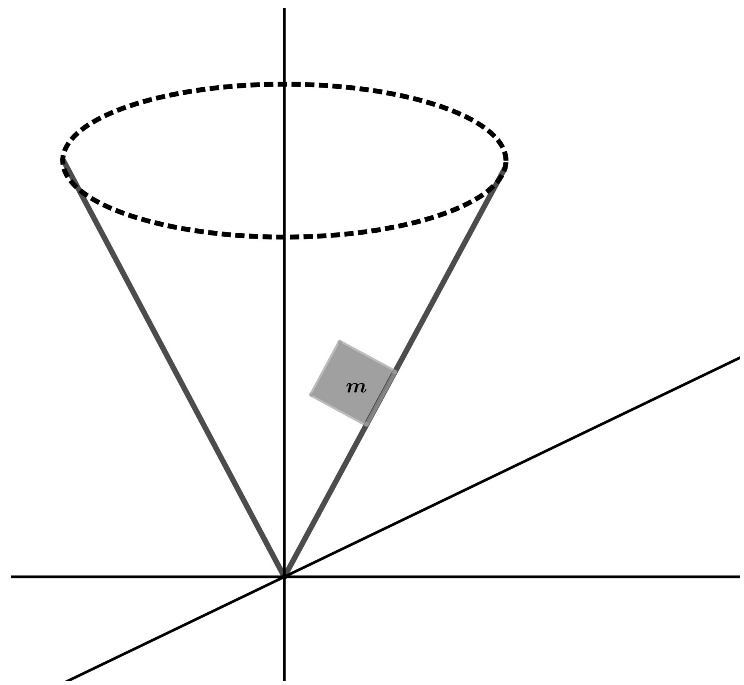

Fig 1: Block constrained to move inside a cone

PART B

Answer any FOUR full questions. **(4x5=20)**

8. Obtain the equation of motion of a block in the inside part of a cone as shown in Fig. 1
9. Obtain the equation of motion of a block suspended vertically from the ceiling of a train moving with constant velocity. The block of mass m is suspended on a spring of force constant k .
10. For a stone of mass m falling down vertically under the influence of gravity, obtain the Hamilton equations of motion.
11. Using the Euler equation, find the extremum of the functional:

$$J = \int_a^b \left[12 x\, y(x) + \left(\frac{d}{dx} y(x) \right)^2 \right] dx$$

12. For the central force field (conservative), we have seen that when we define $u = \frac{1}{r}$ the equations of conservation of angular momentum and energy combine to give us a differential equation: $\frac{d^2 u}{d\phi^2} + u = -\frac{\mu}{\ell^2} \frac{d}{du} V\left(\frac{1}{u}\right)$ where ϕ is the azimuthal (or angular) coordinate, μ is the reduced mass, ℓ is the angular momentum and $V\left(\frac{1}{u}\right)$ the central potential. If the particle in this potential describes an orbit that is a logarithmic spiral $r = k e^{\alpha\phi}$, what form would the central potential take?
13. Obtain the Energy density and energy current for wave on a string described by a Hamiltonian density described in question 6.