Registration Number:
Date & session:

ST.JOSEPH'S UNIVERSITY, BENGALURU -27

M.Sc (PHYSICS) – II SEMESTER
SEMESTER EXAMINATION: APRIL 2024
(Examination conducted in May-June 2024)
PH 8123 – ELECTRODYNAMICS
(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains 3 printed pages and 2 parts.

Some useful Identities:

$$\vec{\nabla}.(f\vec{A})=f(\vec{\nabla}.\vec{A})=\vec{A}.(\vec{\nabla}f)$$

$$\vec{\nabla}x(\vec{A}x\vec{B})=(\vec{B}.\vec{\nabla})\vec{A}-(\vec{A}.\vec{\nabla})\vec{B}+\vec{A}(\vec{\nabla}.\vec{B})-\vec{B}(\vec{\nabla}.\vec{A})$$

In Cylindrical co-ordinates

$$\nabla x\mathbf{v}=\frac{1}{s}[\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}]\hat{s}+[\frac{\partial v_s}{\partial z}-\frac{\partial(v_z)}{\partial s}]\hat{\phi}+\frac{1}{s}[\frac{\partial(s v_\phi)}{\partial s}-\frac{\partial v_s}{\partial\phi}]\hat{z}$$

PART A

Answer any FIVE full questions. Each question carries 7 marks. **(5x7=35)**

1. (a) Considering that the total charge density in a polarised homogenous, linear, isotropic dielectric medium as the sum of bound and free current density, derive Gauss's law for this medium. Obtain the integral form of this law. Obtain the expression for electric displacement $\vec{D}$ and polarisation $\vec{P}$ in terms of applied electric field $\vec{E}$ and explain the physical significance of these terms.
   (b) How does $\vec{\nabla}.\vec{B}=0$ explain non-existence of monopoles? (5+2)

2. Explain the inconsistency in the Ampere's law used in magnetostatics when applied to the case of non-steady currents using the example of charging of a capacitor. How did Maxwell correct this law? Explain the physical significance of the corrected form of this law. (7)

3. (a) From the general form of Maxwell's equations, derive these equations in their integral form.

   (b) A given field $\vec{E}$ produces force $\vec{F}=q\vec{E}$ and according to Newton's law, $\vec{F}=m\vec{a}$ so this field should accelerate the charges. But if the charges are accelerating then the current should increase. However, Ohm's law on the contrary states that constant $\vec{E}$ produces constant 'I'. Is Ohm's law in contradiction to Newton's law? Explain. (5+2)

4. (a) The electromagnetic force per unit volume on the charges in a given volume can be

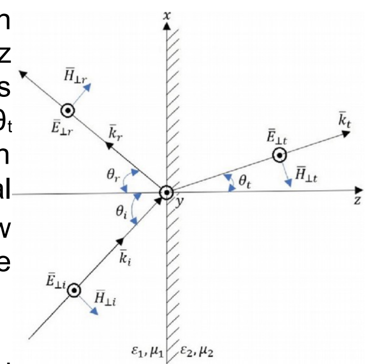
expressed in terms of Maxwell's stress tensor $\boldsymbol{T}$ as $\vec{f} = \nabla . \boldsymbol{T} - \mu_o \epsilon_o \frac{\partial \vec{S}}{\partial t}$ where

$\nabla . \boldsymbol{T} = \epsilon_o [(\nabla . \vec{E})\vec{E} + (\vec{E} . \nabla)\vec{E}] + \frac{1}{\mu_o}[(\nabla . \vec{B})\vec{B} + (\vec{B} . \nabla)\vec{B}] - \frac{1}{2}\nabla(\epsilon_o E^2 + \frac{1}{\mu_o} B^2)$ and $\vec{S}$ is the Poynting vector.

(a) From this, obtain the expression for total force and explain what the elements of stress tensor indicate.

(b) Obtain expression for conservation of momentum in integral and differential form and explain the significance of each term in the equations. (2+5)

5. Suppose an x-y plane forms the boundary between two linear dielectric media at z=0. An incoming monochromatic plane wave of frequency 'ω', travelling in z-direction, polarized perpendicular to the plane of incidence (x-z plane) meets the boundary at an angle θ_i as shown in fig. It gives rise to reflected wave at angle θ_r and transmitted wave at angle θ_t where $\theta_t < \theta_i$ as velocity of wave in medium 2 is less than than in medium 1 $v_2 < v_1$. Assuming that all the three laws of geometrical optics are obeyed and using suitable boundary conditions, show that the Fresnel's equations for this polarization state are $\tilde{E}_{0R} = (\frac{1-\alpha\beta}{1+\alpha\beta})\tilde{E}_{0I}$ and $\tilde{E}_{0T} = (\frac{2}{1+\alpha\beta})\tilde{E}_{0I}$ where $\tilde{E}_{0I}, \tilde{E}_{0R}, \tilde{E}_{0T}$ are the incident, reflected and transmitted amplitudes. Here $\alpha = \frac{\cos\theta_T}{\cos\theta_I}$ And $\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$ .Take $\mu_1 \approx \mu_2 \approx \mu_0$. (7)



6. The electric field of an oscillating dipole is given as $\boldsymbol{E} = \frac{-\mu_o p_o \omega^2}{4\pi} \frac{\sin\theta}{r} \cos[\omega(t - r/c)]\hat{\theta}$.

(a) Calculate the intensity of radiation and discuss its intensity profile.

(b) Also, calculate the total power radiated by this dipole and explain why the blueness of sky in the direction perpendicular to sun's rays is different from that in the direction of rays. (3.5+3.5)

7. (a) What are Gauge transformations? Derive transformation equations for the potentials V, **A**.
(b) Just like retarded potentials for the non-static case, can we have advanced potentials? (6+1)

PART-B

Answer any THREE full questions. Each question carries 5 marks. (3x5=15)

8. The vector potential '**A**' of a magnetic dipole is given as $\vec{A}_{dip}(r) = (\mu_o / 4\pi)(\vec{m} \, x \, \hat{r})/r^2$ where '**m**' is magnetic dipole moment. Show that the magnetic field of this dipole can be written

as: $\vec{B}(r)=\left(\mu_o/4\pi r^3\right)\left[3(\vec{m}.\hat{r})\hat{r}-\vec{m}\right]$

9. The real and imaginary parts of ' $\tilde{k}$ ' are given as:
$k=\omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{\left[1+\left(\frac{\sigma}{\epsilon\omega}\right)^2\right]}+1\right]^{(1/2)}$ and $\kappa=\omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{\left[1+\left(\frac{\sigma}{\epsilon\omega}\right)^2\right]}-1\right]^{(1/2)}$, derive an expression for skin depth of good conductors and hence, calculate the minimum thickness of silver coating required in designing a safe microwave experiment to operate at a frequency of 30 GHz. Given that the resistivity of silver is 1.59×10^{-8} Ω-m and refractive index of silver is 0.15, $\mu_o=4\pi x 10^{-7} H/m$ and $\epsilon_o=8.85 x 10^{-12} F/m$.

10. A long circular cylinder of radius R carries a magnetisation $\vec{M}=k s^2\hat{\phi}$ where k is constant, s is the distance from the axis and $\hat{\phi}$ is azimuthal unit vector. Find the total current and magnetic field due to $\vec{M}$ for points inside and outside the solenoid.

11. The electric field of an electromagnetic wave is given by $\vec{E}=E_o\cos[\pi(0.3x-1000t)]\hat{k}$. Find the magnetic field and the relative dielectric constant of the medium ε_r. If the field is along $\hat{j}$ then calculate the poynting vector $\vec{S}$.