Register number:

Date and session:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
B.SC (MATHEMATICS) - VI SEMESTER
SEMESTER EXAMINATION: April, 2024
(Examination conducted in May/June 2024)

MT 6223: MATHEMATICS VIII

(For current batch students only)

Duration: 2 Hours **Max. Marks:** 60

This paper contains **TWO** printed pages and **THREE** parts.

PART A

ANSWER ANY SIX QUESTIONS **6×2=12**

1. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} xy\, dydx$.
2. Find the volume of the region bounded above by the plane $z = y + 2$ and below by the rectangle $R : 0 \leq x \leq 4, 0 \leq y \leq 2$.
3. Evaluate $\int_1^{e^3} \int_1^{e^2} \int_1^{e} \frac{1}{xyz}\, dx\, dy\, dz$.
4. Evaluate the line integral $\int_C -y\, dx + z\, dy + 2x\, dz$, where C is the helix $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$, $0 \leq t \leq 2\pi$.
5. State Gauss Divergence theorem
6. Compute $U(P, f)$ for the function $f(x) = x^2$ and the partition $P = \{0, \frac{1}{3}, 1\}$ of $[0, 1]$.
7. Compute the norm of the partition $P = \{-2.3, -2, -1.1, 0.6, 1.9, 2.7, 3.1\}$ of $[-2.3, 3.1]$.
8. Give an example of a function f such that f^2 is integrable on some interval but f need not be.

PART B

ANSWER ANY FIVE QUESTIONS **5×6=30**

9. Evaluate the integral $\iint_R (5x - y)\, dA$ by changing to polar coordinates, where R is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 16$ and the lines $x = 0$ and $y = x$.
10. Evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\, dydx$ by changing the order of integration.
11. Find the average value of $f(x, y) = x^2y$ in the triangular region with vertices $(0, 1), (1, 1), (2, 0)$

12. Show that the differential form $2xy\,dx + (x^2 - z^2)\,dy - 2yz\,dz$ is exact and evaluate the integral $\int_{(0,0,0)}^{(1,2,3)} (2xy\,dx + (x^2 - z^2)\,dy - 2yz\,dz)$ over any path from $(0,0,0)$ to $(1,2,3)$.

13. By using the transformation $u = x + 2y$, $v = x - y$, evaluate $\int_0^{\frac{2}{3}} \int_y^{2-2y} (x + 2y)e^{y-x}\,dx dy$

14. State and proof Green's theorem.

15. Verify Stokes' Theorem for the vector field $\mathbf{F} = 2y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$ and surface $S : x^2 + y^2 + z^2 = 1$, $z \geq 0$.

PART C

ANSWER ANY THREE QUESTIONS **3×6=18**

16. Let $f : [a, b] \to \mathbb{R}$ be an integrable function and $k \in \mathbb{R}$ be a constant. Show that $k \cdot f$ is integrable and that $\int_a^b k \cdot f = k \int_a^b f$.

17. Let P be a partition of $[a, b]$ and Q be a refinement of P. Let $f : [a, b] \to \mathbb{R}$ be a bounded function. Show that $L(P, f) \leq L(Q, f)$.

18. Show that a non-constant continuous function on $[a, b]$ is integrable.

19. Let $f : [a, b] \to \mathbb{R}$ be an integrable function that is continuous at a point $c \in [a, b]$. Let $F : [a, b] \to \mathbb{R}$ be defined by $F(x) = \int_a^x f$. Show that $F'(c) = f(c)$.

20. a) Show that the function $f : [0, 1] \to \mathbb{R}$ defined by $f(x) = \begin{cases} x\cos(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is integrable.

    b) True/False: The function $f : [-1, 1] \to \mathbb{R}$ defined by $f(x) = \begin{cases} \frac{x}{x+1} & x \neq -1 \\ 0 & x = -1 \end{cases}$ is integrable. Explain your answer. [3+3]

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