Registration Number:

Date & session:

**ST.JOSEPH'S UNIVERSITY, BENGALURU -27**

**M.Sc (PHYSICS) – II SEMESTER**
**SEMESTER EXAMINATION: APRIL 2024**
**(Examination conducted in May-June 2024)**
**PH 8323 – STATISTICAL PHYSICS**
**(For current batch students only)**

**Time: 2 Hours** **Max Marks: 50**

**This paper contains 3 printed pages and 2 parts**

## PART A

**Answer any FIVE full questions.** **(5x7=35)**

1. Two systems $A$ and $B$, capable of only thermally interacting with each other, form a composite system $A^{(0)}$ that is adiabatically isolated from the rest of the universe. System $A$ is described to have and internal energy between $E$ and $E+\delta E$, while system $B$ has energy between $E'$ and $E'+\delta E'$.
   (a) From general probabilistic considerations, show that on the attainment of equilibrium, the two systems will have $\frac{\partial \ln \Omega(E)}{\partial E}=\frac{\partial \ln \Omega'(E')}{\partial E'}$ where $\Omega(E)$ and $\Omega'(E')$ are the total number of accessible states for the systems $A$ and $B$ respectively.
   (b) The first derivative obtained above, may be denoted by $\beta(E)$ defined as the coldness function. Show that the coldness function is inversely proportional to $E$ and that in order to seek a direct proportionality to $E$, we will produce two more parameters: the temperature $T$ and the entropy $S$. How is $S$ related to $\Omega(E)$ ? [5+2]

2.
   (a) With a figure, state what a Canonical Ensemble is.
   (b) What is the probability distribution for a Canonical Ensemble?
   (c) What is the mathematical expression for the Partition Function for the Canonical Distribution?
   (d) Obtain the mean energy for the Canonical Distribution in terms of its Partition Function.

   [1+1+2+3]

3. Two systems each separately in thermal equilibrium with a heat and particle reservoir of temperature $T$ each having energy $E_r$ and $E_s$ and number of particles $N_r$ and $N_s$ are combined to form one system (in thermal equilibrium with the same heat reservoir as earlier).
   (a) Show that the partition function of the combined system will be the product of the partition functions of the individual systems.

(b) Will the entropy of the combined system too be a product of the individual entropies? Compute the entropy and explain (marks can be given only if there are accompanying equations).

(c) What about the grand potentials in this case – will the grand potential of the combined system be a product of the individual grand potentials? Explain with equations. [3+2+2]

4.

(a) State the equipartition theorem.

(b) Using equipartition theorem, obtain the mean kinetic energy of a molecule in a gas.

(c) What is the partition function of a molecule in an ideal gas and what is the mean internal energy obtained through this partition function? [1+2+4]

5. The quantum mechanical energy for a simple harmonic oscillator to be given by : $\epsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega$ where $n$ is the quantum number, $\hbar$ , the reduced Planck constant and $\omega$ , the natural frequency of the oscillator

(a) Obtain the mean quantum mechanical internal energy of the simple harmonic oscillator.

(b) Using the result from (a) describe a solid in terms of connected simple harmonic oscillators.

(c) Compute the heat capacity of a solid and from this derive the specific heat of solids. [4+2+1]

6.

(a) What is exchange degeneracy?

(b) Describe exchange operator.

(c) Show that there are two fundamental types of particles based on the symmetries of wavefunctions. [1+1+5]

7. For a system of $N$ identical particles partitioned in a manner that the states having energy $\epsilon_i$ have $n_i$ occupancies (assuming the Grand Canonical distribution) we can write the partition function to be: $Z_{GC} = \left(\sum_{n_1} e^{-\beta n_1(\epsilon_1 - \mu)}\right)\left(\sum_{n_2} e^{-\beta n_2(\epsilon_2 - \mu)}\right)\ldots$ where $\mu$ is the chemical potential of the system. Modify this partition function to describe Bosons and obtain the expression for the mean occupancy of Bosons.

## PART-B

**Answer any THREE full questions** **(3x5=15)**

[Constants: $h=6.6\times10^{-34}$ J s (Planck's constant), 1eV = $1.6\times10^{-19}$ J (electron volt to Joules), $c=2.99\times10^{8}$ m/s (speed of light), 1Å = $1\times10^{-10}$m (Angstrom to meters), $k_B = 1.380649\times10^{-23}$ $JK^{-1}$ (Boltzmann constant), $N_A= 6.022 \times 10^{23}$ $mole^{-1}$ (Avogadro Number), e = $1.6\times10^{-19}$ C (electronic charge), $m_{proton}=1.673\times10^{-27}$kg (mass of proton), $m_{electron}=9.109\times10^{-31}$kg (mass of electron), $G=6.674\times10^{-11} m^3 kg^{-1} s^{-2}$ (Gravitational constant), $M_{\odot}=1.9891\times10^{30}$ kg (Solar mass), $R_{\odot}=6.9\times10^{8}$ m, $\sigma=5.67\times10^{-8} Wm^{-2}K^{-4}$(Stefan-Boltzmann constant), $M_{Earth}=5.97\times10^{27}$kg (Mass of Earth), $D_{earth-sun}=1.49\times10^{11}$m (Earth-Sun distance), 1 inch = 2.54 cm, 1AU= $1.496\times10^{11}$ m, 1 ly= $9.461\times10^{15}$ m, 1 pc= $3.086\times10^{16}$m]

8.

(a) A system is made of $3$ particles such that they are confined to a potential providing a each particle a possible $4$ set of states. What is the probability of finding 2 particles in the first excited states?

(b) A particle of mass $m$ is confined to one dimension such that its position is in the domain: $-\ell \leq x \leq \ell$ and is acted upon by the Hooke's Law force (with a force constant $k$) with equilibrium at the center. Draw the phase space diagram for this system such that the particle has an energy between $E$ and $E+\delta E$. [2+3]

9. The mass of air molecules is $4.81 \times 10^{-26}\,\mathrm{kg}$. On a day with the surface temperature of Earth being $36^{\circ}\mathrm{C}$, what is the average height of air molecules close to the surface of Earth? You may take acceleration due to gravity to be $g=9.86\,\mathrm{m.s^{-2}}$. How much would this mean height change if the temperature increased to $38^{\circ}\mathrm{C}$?

10. A certain system, confined to 1 dimension of length $L_x$, has the energy of its individual particles given as: $\epsilon_{xi}=\frac{p_{xi}^2}{2m}$ where $p_{xi}$ is the momentum of the $i^{\mathrm{th}}$ particle along the $x$ direction and $m$ is its mass. Assuming this particle to form a part of a Canonical Ensemble:

(a) Obtain the single particle partition function (using Classical Mechanics arguments).

(b) From the single particle partition function, obtain the average energy of the particle. [3+2]

11. A system consisting of $3$ non-interacting particles, each of which can be in $4$ possible quantum states, each of energy: $0, \epsilon, 2\epsilon, 3\epsilon$. Compute the partition function of the system if the particles are bosons.