Registration Number:
Date & session:

ST JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. (STATISTICS) – 2nd SEMESTER
SEMESTER EXAMINATION: APRIL 2024
(Examination conducted in May / June 2024)
ST 8121 – DISTRIBUTION THEORY
(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains TWO printed pages and ONE part

Note: Scientific calculators are allowed.

PART-A

Answer any FIVE of the following **10 X 5 = 50**

1. A) Define Pareto distribution. Find the mean and quantile function of Pareto distribution.
   B) Find the mean and variance of truncated Poisson distribution (truncated at zero).
   C) For a random variable X with the following probability density function, find the distribution function. (4+4+2)

$$f(x) = \frac{x}{\alpha^2} e^{-\frac{x^2}{2\alpha^2}}, \qquad \alpha > 0, x > 0.$$

2. A) Define symmetric distribution. Prove that normal distribution is symmetric.
   B) If X follows standard Cauchy distribution show that $\frac{1}{X}$ also follows Cauchy distribution.
   C) If X and Y follows Gamma distribution with parameters (n_1, λ) and (n_2, λ)respectively. Find the distribution of Z=X+Y using convolution technique. (3+3+4)

3. A) If $X \sim N(\mu, \Sigma)$ the prove that $CX \sim N(C\mu, C\Sigma C')$, where C is a non-singular matrix.
   B) The joint probability density function of X and Y is

$$f(x,y) = \begin{cases} \frac{6}{5}(x + y^2) & if\ 0 \le x \le 1, 0 \le y \le 1 \\ 0 & Otherwise \end{cases}$$

   Obtain the (i) marginal PDF of X (ii) $E(Y|X = x)$ (5+5)

4. A) Define Gumbel type I bivariate exponential distribution. Obtain the conditional expectation of X given $Y = y$ if (X, Y) follows Gumbel type I bivariate exponential.
   B) Obtain the moment generating function of non-central chi-square distribution. (5+5)

5. A) Define central F distribution. Mention it mean and variance. Show that if X follows t distribution with n degrees of freedom, X^2 follows $F(1, n)$.

   B) Define non-central t and F distribution. Mention their mean and variance (6+4)

6. A) If $\boldsymbol{y} \sim N(0, \sigma^2 I)$ and M is a symmetric idempotent matrix of rank m, prove that

$$\frac{y'My}{\sigma^2} \sim \chi^2(trace\ M)$$

   B) State Fisher Cochran theorem

   C) Show that reciprocal of F distribution is F. (4+2+4)

7. A) Define order Statistics. Derive the PDF of r^{th} order statistic.

   B) If $X_1, X_2, \ldots, X_n$ be a random sample from exponential with mean $\frac{1}{\theta}$, Y_r and Y_s are the r^{th} and s^{th} order statistics, obtain the PDF of $Y_s - Y_r$, $s > r$. (5+5)

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