Registration Number:

Date & session:

ST JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc (DATA ANALYTIC) – 2nd SEMESTER
SEMESTER EXAMINATION: APRIL 2024
(Examination conducted in May / June 2024)
BDA2121 – FOUNDATION OF DATA SCIENCE
(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains 2 printed pages and three parts

PART- A

Answer all the FIVE questions. **[2 X 5 = 10]**

1. For which m and n does the graph $K_{m,n}$ contain an Euler path or an Euler circuit? Explain.
2. Define Gaussian process. What kind of Gaussian distribution is yield in two dimension using identity matrix?
3. State Johnson Lindenstrauss Lemma and give its application in random projection.
4. What is the expected number of edges in $G(n,p)$?
5. Explain the concept of a 2-universal family of hash functions

PART- B

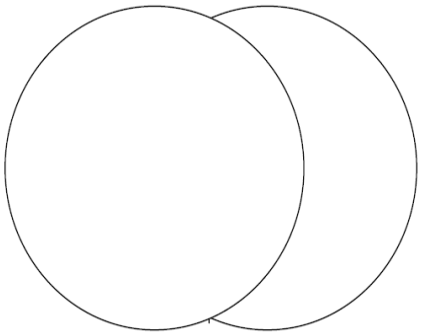
Answer any FIVE questions out of SEVEN questions **[4 X 5 = 20]**

6. Prove that in any group of six people, there will be either three people who know one another or three people do not know one another.
7. Find the number of edges in the complete graph with n vertices with proper explanation.
8. Let X be a random sample from the unit sphere in *d*-dimensions with the origin as center.

(a) What is the mean of this random variable?
(b) What is the variance of (component-wise)?
(c) Given two spheres in space, both of radius one whose centers are distance apart. Show that the volume of their intersection is at most

$$\frac{4e^{-\frac{a^2(d-1)}{2}}}{a\sqrt{d-1}}$$

times the volume of each one.[1+1+2]



9. Let X be a random sample from the unit sphere in *d*-dimensions with the origin as center. Derive the required separation for a pair of dimensional spherical Gaussians, both with the same standard deviation.
10. Examine the conditions for the non- trivial properties of $G(n,p)$?
11. Consider the Markov chain with three states, $S = \{1,2,3\}$, that has the following transition matrix

$$P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

(a) Draw the state transition diagram for this chain.

(b) If $P(X_1 = 1) = P\ (X_2 = 2) = \frac{1}{4}$, find $P(X_1 = 3, X_2 = 2, X_3 = 1)$[1+3]

12. What is the stationary probability of a random walk on an undirected graph?

PART- C

Answer any TWO questions out of THREE questions **[10 X 2 = 20]**

13. For each part below, say whether the statement is true or false. Explain why the true statements are true, and give counterexamples for the false statements.
    (a) Every bipartite graph is planar.
    (b) Every bipartite graph has chromatic number 2.
    (c) Every bipartite graph has an Euler path.
    (d) Every vertex of a bipartite graph has even degree.
    (e) A graph is bipartite if and only if the sum of the degrees of all the vertices is even.[2+2+2+2+2]
14. Explain in detail to determine the number of distinct elements in a Data stream.
15. Find the orthonormal matrix of the matrix of the following matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 2 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$