Register Number:
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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - III SEMESTER

SEMESTER EXAMINATION- OCTOBER 2019

PH 9118 - QUANTUM MECHANICS II

Time-2 1/2 hrs. **Maximum Marks-70**

This question paper has 4 printed pages and 2 parts and a table of constants and integrals

PART A

Answer any FIVE full questions. **(5x10=50)**

1. The Angular Momentum *vector* is given as: $\vec{L}=\vec{r}\times\vec{p}$:
   (a) obtain the components of the Angular Momentum *operator* in Cartesian Coordinates. (**5 Marks**)
   (b) work out the commutation relations between the various Cartesian components (each commutator component should be explicitly worked out) . (**5 Marks**)
2. Using the equations: $\hat{S}_{\pm}\left|s, m_s\right\rangle=\hbar\sqrt{s(s+1)-m_s(m_s\pm 1)}\left|s, m_s\pm 1\right\rangle$ and $\hat{S}_z\left|s, m_s\right\rangle=\hbar m_s\left|s, m_s\right\rangle$ to obtain the eigenstates for a two particle system with spins $s_1=\frac{1}{2}$ and $s_2=1$ respectively.
3. Consider a one dimensional system of two *distinguishable* particles a and b located at points x_1 and x_2 respectively. Obtain the expression for the average of the square of the separation $\langle(x_1-x_2)^2\rangle$ of the two particles.
4. Using the Variational Principle (three dimensions) obtain the ground state energy for the Hydrogen atom. Use a trial wavefunction of the form $\psi_t(\vec{r})=A e^{-br^2}$ where $r=|\vec{r}|$. The expectation value of the Kinetic Energy is given as: $\langle T\rangle=\frac{\hbar^2}{2m}3b$. Assume the Potential energy to be given as $V=\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$ where all the terms are as per usual convention.

5. For a non-degenerate system undergoing perturbation, the first order change in energy is given as: $E_n^{(1)} = \langle \phi_n^{(0)} | \hat{W} | \phi_n^{(0)} \rangle$ where $\hat{W}$ is the perturbing Hamiltonian. The first order change to the eigenstates is given as: $\left| \phi_n^{(1)} \right\rangle = \sum_{m;m\neq n}^{\infty} \frac{\langle \phi_m^{(0)} | \hat{W} | \phi_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} \left| \phi_m^{(0)} \right\rangle$. Obtain the second order change to the energy.
6. Obtain the first order correction to the energy for a system with two fold degeneracy.
7. For time dependent perturbation, we usually assume the time dependent part of the wavefunction to be a linear combination of the eigenstates of the stationary Hamiltonian: $\left| \psi(t) \right\rangle = \sum_n c_n(t) \left| \phi_n \right\rangle$. When we factor out the time evolution of the stationary states, we have the Schrodinger equation reducing to: $i\hbar \frac{d\, b_k}{dt} = \lambda \sum_n b_n(t) e^{-i\omega_{nk} t} \left\langle \phi_k | \hat{W} | \phi_n \right\rangle$. If we perturb b_k , show that the first order perturbed value of b_k : $b_k^{(1)}(t) = \frac{1}{i\hbar} \int_0^t e^{-i\omega_{nk} t'} \left\langle \phi_k | \hat{W} | \phi_n \right\rangle \; dt'$

PART B

Answer any <u>FOUR</u> full questions. (**<u>4x5=20</u>**)

8. The Rodrigues' Formula for Associated Legendre Polynomials is given as:
$$P_\ell^m(z) = (-1)^m (1-z^2)^{m/2} \frac{d^m}{dz^m} P_\ell(z)$$
or, alternatively,
$$P_\ell^m(z) = (-1)^m \frac{(1-z^2)^{m/2}}{2^\ell \ell !} \frac{d^{m+\ell}}{dz^{m+\ell}} (z^2-1)^\ell$$
obtain
   a) $P_4^2(z)$ (**2 Marks**)
   b) $P_4^3(z)$ (**3 Marks**)
9. Two non-interacting particles of mass m are in a one dimensional infinite potential well. Remember, the wavefunction and energies for one particle in an infinite potential are given as: $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$ and $E_n = \frac{\hbar^2}{2m} \frac{n^2 \pi^2}{L^2}$.
   a) What are the ground state wavefunction and energies of the two particles if they are fermions (ignore spin for the particles)? (**3 Marks**)
   b) Verify that the wavefunction you obtained indeed satisfy the Schrodinger equation. (**2 Marks**)
10. Using WKB approximation, obtain the bound state energies of a bouncing ball.
11. Gamow's theory of alpha decay assumes the alpha particle to be trapped in a potential well. Were the alpha particle to tunnel out of such a well, it would experience a repulsive potential of the form: $V(x) = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r}$. Obtain the tunneling probability for the alpha particle.

12. A Simple Harmonic Oscillator (SHO) is perturbed by a anharmonic potential so that the resultant Hamiltonian is given as $H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2 x^2+\lambda x^3$ where $\lambda \ll 1$. The ground state of the SHO is given as: $\psi_0^{(0)}(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$. What is the first order perturbed value of energy?

13. We know that for the time dependent perturbation, the component of the state $|\phi_k\rangle$ to which the state $|\phi_i\rangle$ transitions to is given to a first order as: $b_k^{(1)}(t)=\frac{1}{i\hbar}\int_0^t e^{i\omega_{ki}t'}\langle\phi_k|\hat{W}|\phi_i\rangle dt'$. Furthermore, the transition probability from an initial state $|\phi_i\rangle$ to a final state $|\phi_f\rangle$ can be deduced from this as $P_{if}=\lambda^2|b_f^{(1)}(t)|^2$. For $\hat{W}(t)=\hat{W}_0\sin\omega t$ compute P_{if} .

(Some) Physical Constants

[Constants: **h**=6.626070x10^{-34} J s (**Planck's constant**), **1eV** = 1.6x10^{-19} J (**electron volt to Joules**), **c**=2.99792458x10^{8} m/s (**speed of light**), **1Å** = 1x10^{-10}m (**Angstrom to meters**), **e** = 1.602176x10^{-19} C (**electronic charge**), **ε_0**= 8.85418782x10^{-12}m^{-3}kg^{-1}s^{4}A^{2} (**permittivity of free space**), **m_{proton}**=1.672621898x10^{-27}kg (**mass of proton**), **$m_{electron}$**=9.10938356x10^{-31}kg (**mass of electron**), **$m_{neutron}$**=1.674927471x10^{-27}kg (**mass of neutron**), **a** = 5.029x10^{-10}m (**Bohr radius**), **α** = 1/137 (**Fine Structure Constant**), **G**=6.674x10^{-11}m^{3}kg^{-1}s^{-2} (**Gravitational constant**), **$M_\odot$**=1.9891x10^{30} kg (**Solar mass**), **$R_\odot$**=6.9x10^{8} m (**Sun's Radius**), **σ** = 5.67x10^{-8} W m^{-2} K^{-4} (**Stefan-Boltzmann constant**), **M_{Earth}**=5.97x10^{27}kg (**Mass of Earth**), **$D_{earth-sun}$**=1.49x10^{11}m (**Earth-Sun distance**), 1 inch = 2.54 cm, 1foot=12 inches]

(a) **Table of** (some) **Integrals**

Gamma Function:
$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}\,dt$
$\Gamma(n)=(n-1)!$
$\Gamma(\frac{1}{2}+n)=\frac{(2n)!}{4^n n!}\sqrt{\pi}$

(a) $\int_0^\infty e^{-2bt}\,dt=\frac{1}{2b}$

(b) $\int_0^\infty t\,e^{-2bt}\,dt=\frac{1}{4b^2}$

(c) $\int_0^\infty t^2 e^{-2bt}\,dt=\frac{1}{4b^3}$

(d) $\int_0^\infty t^3 e^{-2bt}\,dt=\frac{3}{8b^4}$

(e) $\int_0^\infty t^4 e^{-2bt}\,dt=\frac{3}{4b^5}$

(f) $\int_0^\infty t^5 e^{-2bt}\,dt=\frac{15}{8b^6}$

(g) $\int_0^\infty t^6 e^{-2bt}\,dt=\frac{45}{8b^7}$

(h) $\int \frac{1}{t^2+b^2}\,dt=\frac{1}{b}\tan^{-1}\left(\frac{t}{b}\right)$

(i) $\int \frac{1}{(t^2+b^2)^2}\,dt=\frac{1}{2b^3}\left(\frac{bt}{(b^2+t^2)}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(j) $\int \frac{1}{(t^2+b^2)^4}\,dt=\frac{1}{16b^7}\left(\frac{15t^5 b+40b^3 5^3+33b^5 t}{(3t^6+9bt^4+9b^3t^2+b^5)}+5\tan^{-1}\left(\frac{t}{b}\right)\right)$

(k) $\int \frac{t^2}{(t^2+b^2)^2}\,dt=\left(-\frac{t}{(2b^2+2t^2)}+\frac{1}{2b}\tan^{-1}\left(\frac{t}{b}\right)\right)$

(l) $\int \frac{1}{(t^2+b^2)^3}\,dt=\frac{3}{8b^5}\left(\frac{5/3b^3t+bt^3}{(b^2+t^2)^2}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(m) $\int \frac{t^2}{(t^2+b^2)^4}\,dt=\frac{1}{16b^5}\left(\frac{bt^5+8/3b^3t^3-3b^5t}{(b^2+t^2)^3}+\tan^{-1}\left(\frac{t}{b}\right)\right)$

(n) $\int \sqrt{a/x-1}\,dx=x\sqrt{a/x-1}+a\ \ \tan^{-1}\left(\sqrt{a/x-1}\right)$

(o) $\int \sqrt{1-ax}\,dx=-\frac{2(1-ax)^{3/2}}{3a}$

(p) $\int \sqrt{1-ax^2}\,dx=\frac{1}{2}x\sqrt{1-ax^2}+\frac{\sin^{-1}\sqrt{a}\,x}{2\sqrt{a}}$

(q) $\int_{-\infty}^{\infty} e^{-\alpha t^2+i\omega t}\,dt=\sqrt{\frac{\pi}{\alpha}}\,e^{-\frac{\omega^2}{4\alpha}}$

(r) $\int_0^\infty t^4 e^{-\alpha^2 t^2}\,dt=\frac{3\sqrt{\pi}}{8\alpha^5}$

(s) $\int_0^\infty t^n e^{-st}\,dt=\frac{n!}{s^{n+1}}$ (Laplace Transform)