Register Number:

Date: 07-01-2021

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE – 27

M.Sc. STATISTICS – I SEMESTER

SEMESTER EXAMINATION – DECEMBER 2020

STA 7220: Statistical Inference – I

Time: 2½hrs **Max:70 Marks**

This question paper has **TWO** printed pages and **TWO** sections

SECTION – A

I Answer any SIX of the following: 6x 3= 18

1. Define statistic and parameter. Give an example for each.
2. State the conditions that need to be satisfied for a distribution to belong to Cramer family, with an example.
3. Let $X_1, X_2, \ldots, X_n$ be a random sample from Bernoulli distribution with parameter p. Examine whether statistic $T = \sum_{i=1}^{n} X_i$ is a complete statistic for p or not?
4. Obtain the sufficient statistic for the parameter of Poisson distribution.
5. Show that convex combination of two unbiased estimators is unbiased. Demonstrate this with an example.
6. What is ancillary statistic? Illustrate with an example.
7. State Rao-Blackwell theorem and explain its utility.
8. Find the Fisher information function contained in a random sample of size n for the distribution with pdf $f(x,\theta) = \theta x^{\theta-1}, 0 < \theta < 1$.
9. Explain the method of moments for estimating the parameters. Give an example where this method fails.

SECTION – B

II Answer any FOUR of the following: 4 x 13 = 52

10. A) Examine whether U (0, θ) is a member of

(i) one parameter exponential family

(ii) pitman family (7)

B) Define k-parameter exponential family. Show that gamma distribution belongs to two-parameter exponential family. (6)

11. A) State and prove Fisher-Neyman factorization theorem (7)

B) Let $X_1, X_2, \ldots, X_n$ be a random sample from shifted exponential with scale and location parameters λ and θ respectively. Find the minimal sufficient statistic for (λ, θ). (6)

12. A) Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential with mean θ examine whether $\bar{x}$ is UMVUE for θ or not (7)

B) State and prove Lehmann-Scheffe theorems (6)

13. A) State and prove Cramer-Rao lower bound inequality. (6)

B) Prove that $(\sum_{i=1}^{n} x_i, \sum_{i=1}^{n} x_i^2)$ is jointly sufficient for (μ, σ^2) for normal distribution. (7)

14. A) Prove that $X_1 + X_2$ is sufficient for the parameter of Poisson distribution. Examine whether $X_1 + 2X_2$ is also sufficient. (6)

B) Explain the procedure for obtaining maximum likelihood estimator. Find the MLE of the parameter of geometric distribution. (7)

15. A) Find the moment estimators of the parameters of beta distribution of first kind with shape parameters α and β. (7)

B) State and prove invariance property of maximum likelihood estimator. (6)