Register Number:

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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE – 27

M.Sc. STATISTICS – I SEMESTER

SEMESTER EXAMINATION – DECEMBER 2020

STA7320: Distribution Theory

Time: 2½hrs **Max:70 Marks**

This question paper has **TWO** printed pages and **TWO** sections

SECTION – A

I Answer any <u>SIX</u> of the following: 6x 3= 18

1. Define quantile function. Find the same for exponential distribution with mean θ.
2. Let X be a non-negative integer valued random variable with probability generating function (PGF) $P_X()$.Then show that $\int_0^1 P_X(t)dt = E\left(\frac{1}{X+1}\right)$.
3. Prove that the distribution function is non decreasing.
4. Prove that $V(Y/X) = \mathrm{E}(Y^2/X) - (\mathrm{E}(Y/X))^2$.
5. List at least three properties of the variance covariance matrix of multivariate normal random variable.
6. Let A be a square matrix. Let Y be a vector random variable with $E(Y) = \mu$ and $V(Y) = \Sigma$. Then show that $E(Y^T Ay) = trace(A\Sigma) + \mu^T A\mu$.
7. State and prove reciprocal property of F distribution.
8. Let $X_1, X_2, \ldots, X_n$ be a random sample from U (0,1). Find the distribution of r^th order statistic Y_r.
9. Define order statistic and explain its importance.

SECTION – B

II Answer any FOUR of the following: 4 x 13 = 52

10. A) Find mean and variance of Poisson distribution truncated at zero. (6)

B) Decompose the following distribution function into discrete and continuous distribution functions.

$$F(x) = \begin{cases} 0 & if\ x < 0 \\ \frac{1}{4} + \frac{x}{4} & if\ 0 \le x < 1 \\ \frac{1}{2} + \frac{x}{4} & if\ 1 \le x < 2 \\ 1 & if\ x \ge 2 \end{cases}$$ (7)

11. A) Let $P_X()$ and $P_Y()$ be the PGF of X and Y respectively. Then prove that $P_X(t) = P_Y(t)$ if and only if X and Y have the same probability distribution. (6)

B) Let $X \sim N_p(\mu, \Sigma)$ and $C_{p\times p}$ be a non-singular matrix. Prove that $CX \sim N_p(C\mu, C\Sigma C')$ (7)

12. A) Let joint pdf of X and Y be

$$f(x,y) = \begin{cases} 4xye^{-(x^2+y^2)} & if\ x \ge 0, y \ge 0 \\ 0 & Otherwise \end{cases}$$

Examine whether X and Y are independent or not (6)

B) Let (X, Y) be a bivariate Gumbel type I distribution. Find E(X/Y). (7)

13. A) Derive the expression for moment generating function (MGF) of non-central chi-square distribution. (8)

B) If X follows t distribution with n degrees of freedom, find the probability distribution of X^2 (5)

14. A) Let X and Y be the two independent exponential random variables with common mean $\frac{1}{\theta}$. Using convolution theorem find the density of X+Y. (6)

B) If $Y \sim N_p(0, \sigma^2 I)$ and M is a symmetric idempotent matrix of rank m, then prove that $\frac{Y'MY}{\sigma^2} \sim \chi^2(trace(M))$. Where I is the identity matrix (7)

15. A) Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution. Derive the joint probability density function of r^{th} and s^{th} order statistics. (6)

B) Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential with mean $\frac{1}{\theta}$. Let Y_r and Y_s are the r^{th} and s^{th} order statistics. Show that Y_r and $Y_s - Y_r$ are independent for any s>r. (7)