ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE – 27

M.Sc. STATISTICS – I SEMESTER

SEMESTER EXAMINATION – DECEMBER 2020

STA7420: Mathematical Analysis

Time: 2½hrs   **Max:70 Marks**

This question paper has **TWO** printed pages and **TWO** sections

SECTION – A

I   **Answer any SIX of the following:**   **6x 3= 18**

1. Distinguish between supremum and infimum of a set. For any set A, Prove that

$$\inf(A) = -\sup(-A).$$

2. Define open set. Prove or disprove: A finite intersection of open sets is open

3. If [x] represents the integer part of x, Evaluate

$$\int_0^2 \frac{1}{1+x^2}\, d(x^2 + [x]).$$

4. Discuss the convergence of the integral.

$$\int_1^\infty \frac{\sin^2 x}{x^4}\, dx$$

5. Derive Legendre Duplication Formula.

6. Verify whether $F(x)$ converges uniformly in the interval $1 < x < 2$.

$$F(x) = \int_0^\infty x\, e^{-xt}\, dt$$

7. Evaluate the limit

$$\lim_{n\to\infty} \frac{1}{n}\, \sqrt[n]{n!}$$

8. Distinguish between pointwise convergence and uniform convergence of a sequence of functions.

9. Prove that every uniformly convergent sequence of functions is Cauchy.

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SECTION – B

II   **Answer any FOUR of the following:**   **4 x 13 = 52**

10.

A) Prove that a monotonic sequence converges if and only if it is bounded.   (6)

B) State and prove the existence of Riemann-Stieltjes integral   (7)

11.

A) Establish the change of variable property of Riemann-Stieltjes integral   (6)

B) If f is monotonic increasing on $[a; b]$, and if α is continuous on $[a; b]$, then prove that there exists a point $c \in [a, b]$ such that

$$\int_a^b f(x)d\,\alpha(x) = f(a)\int_a^c d\,\alpha(x) + f(b)\int_c^b d\,\alpha(x)$$

(7)

12.

A) State and prove Abel's test for the convergence of improper integrals   (7)

B) Define Beta and Gamma integrals. Establish the relationship between them. (6)

13.

A) Find the extreme points of the function $x^4 + y^4 - x^2 - y^2 + 1$ (6)

B) Mention the utility of Lagrange's multiplier technique? Using this technique or otherwise, find the maximum volume of rectangular parallelepiped of surface area a^2.   (7)

14.

A) Evaluate the double integral

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (1 - x^2 - y^2)dx\,dy$$

(6)

B) Suppose $\{f_n, n \geq 1\}$ converges uniformly to f on [a, b] and α be monotonically increasing on $[a, b]$. If $f_n \in R(\alpha)$ on [a, b] then prove that $f \in R(\alpha)$ on $[a, b]$. (7)

15.

A) State and prove Weierstrass test for uniform convergence of functions.   (6)

B) Prove that a complex valued function is analytic if and only if its real and imaginary parts satisfy Cauchy-Riemann equations.   (7)