Register Number:

DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc. MATHEMATICS – VI SEMESTER
SEMESTER EXAMINATION: APRIL 2018
MT6115 : MATHEMATICS - VII

Time- 2 ½ hrs. **Max Marks-70**

This question paper has three parts and two printed pages.

I. Answer any five from the following **5 X 2 =10**

1. Solve $\frac{dx}{y^2}=\frac{dy}{x^2}=\frac{dz}{x^2y^2z^2}$.
2. Form the partial differential equation by eliminating arbitrary constants a and b from $z = ax + by + a^2 + b^2$.
3. Solve $pe^y = qe^x$.
4. Solve $(D^2 - 2DD' + D'^2)\, z = 0$.
5. Write the expression for arc element and volume element of cylindrical co-ordinates.
6. Show that the subset $W = \{(x_1, x_2, x_3) : x_1 + x_2 + x_3 = 0\}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$.
7. Show that the set $B = \{(1,1,0)\ ,\ (1,0,1)\ ,\ (0,1,1)\}$ is a basis of the vector space $V_3(R)$.
8. Let $T : V \to W$ be a non-singular linear map, then prove that $T^{-1} : W \to V$ is also a non-singular linear map.

II. Answer any seven from the following **7 X 6 = 42**

9. Verify the condition for integrability and solve the differential equation $(2x^2 + 2xy + 2xz^2 + 1)\,dx + dy + 2z\,dz = 0$.

10. Form a partial differential equation by eliminating arbitrary constants f and g from $z=\frac{1}{y}[f(x+ay)+g(x-ay)]$.

11. Solve $\frac{y-z}{yz}p+\frac{z-x}{zx}q=\frac{x-y}{xy}$.

12. Solve $p(1+q)=zq$.

13. Solve $px+qy=pq$ using Charpit's method.

14. Find the general solution of $2r-s-3t=5e^{x-y}$.

15. A lightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y=y_0\sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest in this position, find the displacement $u(x,t)$.

16. Derive arc element and volume element in spherical co-ordinate system.

17. Express the vector $\vec{A}=z\,\hat{i}-2x\,\hat{j}+y\,\hat{k}$ in terms of cylindrical co-ordinates.

III. Answer any three from the following **3 X 6 =18**

18. Prove that the intersection of any two subspaces of a vector field $V(F)$, is also a subspace of $V(F)$. Is union of two subspaces a subspace? Justify.

19. An ordered set $\{\alpha_1,\alpha_2,........\alpha_n\}$ of non-zero n vectors of a vector space $V(F)$ with $\alpha_1\neq 0$, is linearly dependent if and only if one of the vectors say α_k where $2\leq \mathrm{k}\leq \mathrm{n}$, is a linear combination of its preceding ones.

20. Find the matrix of the linear transformation $T:V_2(R)\rightarrow V_3(R)$ defined by $T(x,y)=(2y-x,y,3y-3x)$ relative to the bases $B_1=\{(1,1),(-1,1)\}$ and $B_2=\{(1,1,1),(1,-1,1),(0,0,1)\}$.

21. State and prove rank nullity theorem.