Register Number:

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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE
M.Sc. MATHEMATICS – III SEMESTER
SEMESTER EXAMINATION: NOVEMBER 2020
MT9218: CLASSICAL AND CONTINUUM MECHANICS

Time- 2 ½ hrs.

Max Marks-70

The paper contains TWO pages.

Answer any SEVEN full questions. Each carrying 10 marks.

1. a) Derive the expression for velocity and acceleration in plane polar co-ordinate system.

   b) Find the velocity of the particle at $\theta = 30^0$, given $r = 5\cos 2\theta \ (m)$, $\dot{\theta} = 3t^2 \ (rad/\sec)$ and $\theta_0 = 0$.

   (6+4)

2. a) Derive the expression for centrifugal force and Coriolis force.

   b) A projectile of mass 5kg, in its course of motion explodes on its own into two fragments. One fragment of mass 3kg falls at three fourth of the range R of the projectile. At what distance does the other fragment fall from the point of launching?

   (7+3)

3. a) Derive the expression for conservation of linear momentum for the system of particles.

   b) Define holonomic and non-holonomic constraints.

   (6+4)

4. a) Show that every conservative system, either holonomic or non-holonomic has a constant Hamiltonian function.

   b) Define Poison's bracket of two function.

   c) If $Q = q^\alpha \sin\beta p$, $P = q^\alpha \sin\beta p$ and if the following transformation is canonical, find the value of α and β.

   (4+2+4)

5. a) If $D = det(a_{ij})$. Verify that $\varepsilon_{ijk}\varepsilon_{pqr}D = \begin{vmatrix} a_{ip} & a_{iq} & a_{ir} \\ a_{jp} & a_{jq} & a_{jr} \\ a_{kp} & a_{kq} & a_{kr} \end{vmatrix}$.

   Also deduce the following results:

   i) $\varepsilon_{ijk}\varepsilon_{pqr} = \begin{vmatrix} \delta_{ip} & \delta_{iq} & \delta_{ir} \\ \delta_{jp} & \delta_{jq} & \delta_{jr} \\ \delta_{kp} & \delta_{kq} & \delta_{kr} \end{vmatrix}$

   ii) $\varepsilon_{ijk}\varepsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$

b) Given in x_i- system a vector $\vec{a}$ has components $a_1 = -1, a_2 = 0, a_3 = 1$ and a tensor $\vec{A}$ has its matrix $[a_{ij}] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$, the x'_i- system is obtained by rotating the x_i-system about the x_1-axis through an angle 45^0 in the sense of the righthanded screw. Find the components of $\vec{a}$ and $\vec{A}$ in x'_i- system. (5+5)

6. a) Prove that if $\vec{A}$ is a skew tensor then there exists a vector $\vec{w}$ of $\vec{A}$ such that $\vec{A}\vec{u} = \vec{w} \times \vec{u}$ for every vector $\vec{u}$, where $\vec{w}$ is called a dual of vector of skew tensor $\vec{A}$.
   b) State and prove Stokes theorem for a tensor. (4+6)

7. a) Find the velocity and acceleration field in material and spatial form for the equation

$$x_1^0 = x_1 \cos \alpha t - x_2 \sin \alpha t$$
$$x_2^0 = x_1 \sin \alpha t + x_2 \cos \alpha t \,.$$

   b) For the deformation defined by following the equations $x_1^0 = \frac{1}{2}\left(x_1^2 + x_2^2\right)$, $x_2^0 = \tan^{-1}\left(\frac{x_2}{x_1}\right)$ and $x_3^0 = x_3$, find $F \text{ and } F^{-1}$. Also show that the deformation is isochoric. (5+5)

8. a) Obtain the expression for strain displacement relation in the spatial description form.
   b) Find the path and stream lines for the motion given by

$$v_1 = \frac{x_1}{1+t}, v_2 = \frac{2x_2}{1+t} \text{ and } v_3 = \frac{3x_3}{1+t}.$$

(4+6)

9. a) Derive the expression for Reynold's transport formula.
   b) Show that the motion of a continuum in circulation is preserved if and only if the acceleration is an irrotational vector. (6+4)

10. (a) Derive the expression for conservation of mass in material form.
    (b) Derive the expression for conservation of energy. (3+7)

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