Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. MATHEMATICS - II SEMESTER
SEMESTER EXAMINATION: APRIL 2018

MT 8214: COMPLEX ANALYSIS

Time- 2 1/2 hrs **Max Marks-70**

This paper contains TWO printed pages

Answer any SEVEN questions from the following.

1. a) If 'f' is analytic over a simply connected domain D and C is a simple closed curve that lies inside D, then Show that $f^{(n)}(a) = \frac{n!}{2\pi i} \oint_c \frac{f(z)}{(z-a)^{n+1}} dz,$ where (n=1,2,3……..)

   b) Evaluate $\oint_c \frac{3z^2+4z-1}{(z^2+4)(z^2+1)} dz,$ where $c: |z| = 3$. **(5+5)**

2. State and Prove Cauchy's Theorem for a rectangle. **(10)**

3. a) Show that "Suppose $f(z)$ is analytic at z_0 then $f(z)$ has a zero of order 'm' at z_0 iff $f(z)$ can be written in the form $f(z) = (z-z_0)^m g(z)$, where $g(z)$ is analytic at z_0 and $g(z_0) \neq 0$."

   b) Show that "A complex function $f(z)$ has a pole of order 'm' at z_0 iff $f(z)$ can be written in the form $f(z) = \frac{\phi(z)}{(z-z_0)^m}$ where $\phi(z)$ is analytic in the neighbourhood of z_0 and $\phi(z_0) \neq 0$." **(5+5)**

4. a) State and Prove Taylor's Theorem.

   b) Expand $f(z) = \frac{z+1}{(z+2)(z+3)}$ in a laurentz series valid for

   (i) $|z| > 3$ (ii) $2 < |z| < 3$ **(5+5)**

5. Let $'R'$ be the Radius of convergence of the Power series

$f(z) = \sum_{n=0}^{\infty} a_n z^n$, then Prove the following:

(i) The derived power series $\sum_{n=0}^{\infty} a_n z^{n-1}$ has the same Radius of convergence as the original power series $\sum_{n=0}^{\infty} a_n z^n$.

(ii) The sum function $f(z)$ is analytic for $|z| < R$.

(iii) The sum function $f(z)$ is infinitely differentiable over $|z| < R$. **(10)**

6. a) Define Removable singularity and Pole, give examples for each.
   b) Define Residue and discuss the Residue of the following function at each of the pole, $f(z) = \dfrac{e^z}{z^2(z-5)^3}$.
   c) Derive the formula to find the Residue at the pole of order m. **(3+5+2)**

7. Evaluate $\int_{-\infty}^{\infty} \dfrac{e^{ax}}{e^x + 1}$, $0 < a < 1$. **(10)**

8. State and Prove Hadamard's three circle Theorem**.** **(10)**

9. a) State and Prove Maximum modulus Theorem.
   b) State and Prove Weierstrass factorization Theorem. **(7+3)**

10. State and Prove Poisson's integral Formula. **(10)**