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Register Number:

DATE:

**ST. JOSEPH’S COLLEGE (AUTONOMOUS), BANGALORE-27**

**M.Sc. MATHEMATICS – II SEMESTER**

**SEMESTER EXAMINATION: APRIL 2018**

MT 8314: Functional Analysis

Time- 2 ½ hrs Max Marks-70

This paper contains \_1\_\_printed pages and four parts

**Answer any 7 questions out of the following 10 questions.**

1. a) Define a Banach space .

b) Show that is a Banach space , with the norm defined by    
 where . [2+8]

1. Let be a closed linear subspace of a normed linear space . Let be the quotient space is defined by , .Prove that is a normed linear space and is a Banach space if is a Banach space. [10]
2. Show that the normed linear space of all continuous linear transformations from , is a complete normed linear space if is complete. [10]
3. a) If *B* and *B'* are Banach spaces, and if *T* is a continuous linear transformation of   
    *B* onto *B',* then the image of each open sphere centered on the origin in *B*   
    contains an open sphere centered on the origin in *B'*.  
   b) State and prove the Open Mapping theorem. [7 + 3]
4. State and prove the uniform boundedness theorem. [10]
5. a) Define a conjugate space of a Hilbert Space.

b) State and Prove the Grahm Schmidt Ortho-normalisation process. [3+7]

1. a) Prove that any 2 Orthonormal sets in a Hilbert Space H have the same cardinality.   
   b) Every Orthonormal set in a Hilbert Space is contained in a complete Orthonormal   
    set? [3+7]
2. a)An operator T on H is unitary iff it is an isometric isomorphism of H onto itself.

b) Show that the unitary operators on H form a group. [4+6]

1. a) Prove that the inner product and the norm are continuous.

b) State and prove the Bessel’s Inequality for a finite Orthonormal Set. [5+5]

10. a) Define Isometric Isomorphism?

b) State and Prove the Riesz Representation Theorem.

c) If T is an operator on H, then T is normal iff it is real and imaginary parts   
 commute. [1+5+4]