Register Number:
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ST.JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS – II SEMESTER

SEMESTER EXAMINATION – APRIL 2018

PH 8115 : ELECTRODYNAMICS

Time: 2.5 hours **Maximum Marks:70**

This question paper contains 2 parts and 3 printed pages.

Some useful Identities:

$$\vec{\nabla}.(\vec{A}\,x\,\vec{B})=\vec{B}.(\vec{\nabla}\,x\,\vec{A})-\vec{A}.(\vec{\nabla}\,x\,\vec{B})$$

$$\vec{\nabla}\,x(\vec{A}\,x\,\vec{B})=(\vec{B}.\vec{\nabla})\vec{A}-(\vec{A}.\vec{\nabla})\vec{B}+\vec{A}(\vec{\nabla}.\vec{B})-\vec{B}(\vec{\nabla}.\vec{A})$$

In Spherical polar co-ordinates

$$\nabla t=\frac{\partial t}{\partial r}\hat{r}+\frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\theta}+\frac{1}{r\sin\theta}\frac{\partial t}{\partial\varphi}\hat{\varphi}$$

In Cylindrical co-ordinates

$$\nabla\,x\,\mathbf{v}=\frac{1}{s}[\frac{\partial v_z}{\partial\varphi}-\frac{\partial v_\varphi}{\partial z}]\hat{s}+[\frac{\partial v_s}{\partial z}-\frac{\partial(v_z)}{\partial s}]\hat{\varphi}+\frac{1}{s}[\frac{\partial(s\,v_\varphi)}{\partial s}-\frac{\partial v_s}{\partial\varphi}]\hat{z}$$

All bold letters also denote vectors where ever relevant.

Part-A

Answer any 5 questions. Each question carries 10 marks. **(10x5=50)**

1. a) Suppose there did exist magnetic monopoles, how would you modify Maxwell's equations and the force law to accomodate them?
   b) Starting with the Biot-Savart's law for the general case of a volume current, derive and show that the curl of magnetic field is in agreement with the Ampere's law. (4+6)

2. What was the inconsistency with Ampere's law that led Maxwell to theorize that changing electric field produces magnetic field?Derive the corrected form of the law as given by Maxwell and justify it with the example of charging of capacitor. Explain why did he call the time varying field term as displacement current. (10)

3. Suppose an x-y plane forms the boundary between two linear media. An incoming monochromatic plane wave of frequency 'ω', polarized in the plane of incidence (x-z plane) meets the boundary at an arbitrary angle θ_I. It gives rise to reflected wave at angle θ_R and transmitted wave at angle θ_T where $\theta_T < \theta_I$. Assume that all the three laws of geometrical optics are obeyed. Using appropriate boundary conditions, arrive at the two Fresnel's equations for this polarization state. (10)

4. Reformulate Maxwell's equations in terms of the potentials for the time-dependent configuration and then use Coulomb's gauge to simplify them. In what way is this gauge helpful?What are the disadvantages of this gauge? (10)

5. The retarded potentials of a moving(accelerating) point charge particle are given as : $V=\frac{1}{4\pi\epsilon_o}\frac{qc}{(rc-\vec{r}.\vec{v})}$ and $\vec{A}=\frac{\vec{v}}{c^2}V(\vec{r},t)$ where V, $\vec{A}$ are scalar and vector potentials respectively, $\vec{v}$ is the velocity of the charge at the retarded time and $\vec{r}$ is the vector from the retarded position to the field point. Also the gradient of this potential V is given as $\nabla V=\frac{1}{4\pi\epsilon_o}\frac{qc}{(rc-\vec{r}.\vec{v})^3}[(rc-\vec{r}.\vec{v})\vec{v}-(c^2-v^2+\vec{r}.\vec{a})\vec{r}]$.
Show that the time derivative of vector potential is given as: $\partial\vec{A}/\partial t=\frac{1}{4\pi\epsilon_o}\frac{qc}{(rc-\vec{r}.\vec{v})^3}[(rc-\vec{r}.\vec{v})(-\vec{v}+\frac{r\vec{a}}{c})+\frac{r}{c}(c^2-v^2+\vec{r}.\vec{a})\vec{v}]$. { Substitute $\vec{u}=c\hat{r}-\vec{v}$ and $\frac{\partial t_{retarded}}{\partial t}=\frac{cr}{\vec{r}.\vec{u}}$ wherever required.} (10)

6. The electric field of a dipole oscillating with frequency ω is given as : $\boldsymbol{E}=\frac{-\mu_o p_o\omega^2}{4\pi}\frac{\sin\theta}{r}\cos[\omega(t-r/c)]\hat{\theta}$ and the magnetic field is given as : $\boldsymbol{B}=\frac{-\mu_o p_o\omega^2}{4\pi c}\frac{\sin\theta}{r}\cos[\omega(t-r/c)]\hat{\varphi}$. Find the energy radiated by this oscillating dipole and calculate the power radiated. Also, explain the intensity profile. (10)

7. a) Show how the velocity 4-vector for proper velocity transforms as we go from one inertial frame to another. Given that the transformation matrix M for transforming from one inertial frame to another is: $M=\begin{pmatrix}\gamma & -\gamma\beta & 0 & 0\\ -\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$ and hence explain the concept of proper velocity and ordinary velocity.
b) Write the relativistic momentum in terms of a 4-vector. What do temporal and spatial components denote? Find the scalar four dimensional product of momentum with itself and interpret the result.
(6+4)

Part-B

Answer any 4 questions. Each question carries 5 marks. (4x5=20)

8. What current density would produce the vector potential, $\vec{A}=k\hat{\varphi}$ in cylindrical co-ordinators (where k is a constant) ?

9. A solid sphere of radius R has a charge density ρ given by $\rho=\rho_o(1-\frac{ar}{R})$ whre r is the radial co-ordinate ρ_o, a and R are positive constants. If the magnitude of electric field at $r=R/2$ is 2.5 times that at $r=R$, then find the value of a .

10. The vector potential corresponding to a charge and current distribution is given as $\vec{A}=\frac{-1}{4\pi\epsilon_o}\frac{qt}{r^2}\hat{r}$ where symbols have their usual meaning. If this potential under a valid gauge transformation changes to $\vec{A}'=0$ then what is the corresponding change in scalar potential (i.e. V-V')?

11. The electric and magnetic fields in a conductor have plane wave solutions but the wavenumber $\tilde{k}$ is a complex quantity and is given as $\tilde{k} = k + i\kappa$ where

$$k=\omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{\left(1+\frac{\sigma}{\epsilon\omega}\right)}+1\right]^{(1/2)} \qquad \kappa=\omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{\left(1+\frac{\sigma}{\epsilon\omega}\right)}-1\right]^{(1/2)}$$

a) Show that in a good conductor, the magnetic field lags the electric field by 45^0 .

b) In poor conductors is $\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$ and in good conductors is $\sqrt{\frac{2}{\omega\mu\sigma}}$. (2+3)

12. A 125-turn rectangular coil of wire with sides of 25 and 40 cm rotates about a horizontal axis in a vertical magnetic field of magnitude 3.5 mT. How fast must this coil rotate for the induced emf to reach 5 volts?

13. Event A happens at point x_A=5, y_A= 3 and z_A= 0 and at time t_A given by ct_A = 15. Event B occurs at x_B =10, y_B = 8 and z_B = 0 and at time t_B given by ct_B = 5, both in system S.
i) What is the invariant interval between A and B ?
ii) Is there an inertial frame in which they occur simultaneously? If so, find it's velocity relative to frame S.
iii) Is there an inertial frame in which they occur at the same point? If so, find it's velocity relative to frame S.