Date:
Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.Sc. MATHEMATICS - III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2021
(Examination conducted in January-March 2022)

MT 9118- FUNCTIONAL ANALYSIS

Time- 2 ½ hrs Max Marks-70

This question paper contains ONE printed page and ONE part

Answer any 7 questions

1.
   i. State and Prove parallelogram law of the norm induced by an inner product space.
   ii. Let X_0 be a finite dimensional proper subspace of a normed linear space X. Then, prove that there exists $x \in X$ such that $||x|| = 1, dist(x, X_0) = 1$. [5+5]

2. State and prove Gramm Schmidt Orthogonalization. [10]

3. State and prove Minkowski's inequality for l^p $where$ $1 < p < \infty$. [10]

4. Let X be a normed linear space. Then, show that X is a Banach space iff every absolutely convergent series of elements of X is convergent. [10]

5. If X_0 is complete subspace of a normed linear space X and $X/_{X_0}$ is a Banach space, then show that X is a Banach space. [10]

6. State and prove Riesz representation Theorem. [10]

7.
   i. Show that the bounded opereator $\mathcal{B}(X,Y)$ is a subspace of linear operator $L(X,Y)$.
   ii. State and prove Riesz -Fischer Theorem. [5+5]

8. Let X be a Hilbert space and E be an orthonormal basis of X. Then, show that E is a basis iff X is finite dimensional. [10]

9. Let X be a normed linear space and Ω be dense subset of X. Then, show that X is linearly isometric with a subspace of $l^\infty(\Omega)$. [10]

10. State and prove Open mapping Theorem. [10]