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Register Number:

Date:5-03-2022

**ST. JOSEPH’S COLLEGE (AUTONOMOUS), BANGALORE –560 027**

M.Sc. STATISTICS – I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2021

(Examination conducted in March 2022)

 **ST 9120 - Stochastic Processes**

**Time: 2 ½ hrs Max Marks-70**

This question paper has **TWO** printed pages and **TWO** parts

**Part A**

**Answer any SIX of the following 6x 3= 18**

1. Define a Markov Chain (MC). Show that sequence of independent identically

distributed (i.i.d.) random variables are MC.

1. Define period of a state. Give an example of a Markov chain with a state

having period 2.

1. Prove that $i⟷j  $and $j⟷k$ $⇒i⟷k$.
2. Check whether a Markov chain with states space S={1, 2, 3} and following TPM is an irreducible or not.

$ P=\left[\begin{matrix}\frac{1}{3}&0&\frac{2}{3}\\0&\frac{1}{4}&\frac{3}{4}\\\frac{1}{2}&0&\frac{1}{2}\end{matrix}\right]$

1. Show that the inter occurrence times of a Poisson Process follows iid Exponential distribution.
2. Define birth and death process. Illustrate with an example.
3. Define following terms
4. Martingale
5. Sub martingale
6. Super martingale
7. Explain Galton-Watson branching process? Give any two applications.

**Part B**

**Answer any FOUR of the following 4x 13= 52**

1. A) Briefly explain classification of stochastic processes with example for each classification. (4)

B) Write a note on application of Markov chain. (3)

 C) Prove that if state $i$ is recurrent and $i⟷j $then $j$ is also recurrent. (6)

1. A) State and prove Chapman-Kolmogorov equation of a homogeneous MC. Mention its one application. (6)

 B) Define persistent and transient state. If state i of a MC is persistent and $i⟷j  $

then show that state j is also persistent. (7)

1. A) Prove that state $j$ is recurrent if and only if $\sum\_{n=1}^{\infty }p\_{jj}^{\left(n\right)}=\infty $ (5)

B) Stating the postulates of a Poisson Process X(t) show that,

 $P\left[X\left(t\right)=n\right]=\frac{e^{-λ}λ^{n}}{n!}$ (8)

1. A) Derive the birth and death process from a set of postulates to be stated. Obtain

the difference differential equation. (8)

B) Define continuous time Markov chain. State and prove Chapman-Kolmogorov

 equation for continuous time Markov chain (5)

1. A) State and prove elementary renewal theorem. (9)

B) Let $\left\{N\left(t\right)\_{ }\right\}$ be a Poisson process then prove that $N\left(s\right)/N(t)$ follows Poisson distribution if $s>t.$ (4)

1. A) Derive mean and variance of a Galton-Watson branching process (Xn), if the

offspring random variable has mean m and variance $σ^{2}$. (10)

B) Let $\left\{Z\_{i}:i=1,2,3,..\right\}$ be a sequence of iid random variables with mean 0 and let $X\_{n}=\sum\_{i=1}^{n}Z\_{i}$. Prove that $\left\{X\_{n}:n\geq 0\right\}$ is a Martingale. (3)