Register Number:
DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27

M.Sc. PHYSICS - I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2021

(Examination conducted in January-March 2022)

PH 7120/7121 - CLASSICAL MECHANICS

Time-2 1/2 hrs. Max Marks-70

This question paper has 2 printed pages and 2 parts

Part A

Answer any 5 questions **(5x10=50)**

1. For a single particle moving in one dimension
   (a) starting from the definition of kinetic energy show that defining a potential $V(x)$ such that $F(x)=-\frac{dV}{dx}$ leads to conservation of total energy.
   (b) From this, define conservative forces. [8+2]
2. What are integrals of motion? How many independent integrals of motion will a system with n degrees of freedom have? Give and example of the determination of integrals of motion for a system.
3. For a particle moving in Central Force Potential (conservative): $V(|\vec{\boldsymbol{r}}|)$ show that there is conservation of total angular momentum.
4. Integration of the radial component of the Lagrange equation for a particle moving in a Central Force Potential gives us the conservation of total energy equation as: $E=\frac{1}{2}\mu\left(\dot{r}^2+\frac{\ell^2}{\mu^2 r^2}\right)+V$ where, each of the terms have their usual meaning. Show that the component $\frac{\ell^2}{m^2 r^2}$ arises from a conservative-like force.
5. Explain:
   (a) What is Legendre Transformation?
   (b) How does Legendre Transformation lead to obtaining the Hamiltonian from the Lagrangian? [4+6]

6. From basic arguments of three blocks of mass connected by springs, obtain the Lagrangian Density for a continuous medium like a string.
7. For a system rotating about an axis directed in an arbitrary direction, obtain the transformation relation for the rate of change of a vector $\vec{A}$ with respect to time from the rotating frame to that in the inertial frame – in other words, show that: $\left.\frac{d\vec{A}}{dt}\right|_{\text{inertial}} = \left.\frac{d\vec{A}}{dt}\right|_{\text{rot}} + \hat{\boldsymbol{\Omega}} \times \vec{A}$.

Part B

Answer any 4 questions **(4x5=20)**

8. A bead of mass m is constrained to move on a vertical parabolic path:
   (a) Write down the equation of constraint of the bead
   (b) Express the potential energy of the bead in terms of the generalized coordinate. [2+3]
9. A stone (of mass m) is set into motion in a vertical circle.
   (a) Compute the Lagrangian of the system
   (b) From the Lagrangian, work out its equation of motion. [2.5+2.5)
10. Compute the optimal path that makes the following integral stationary: $J = \int_{x_1}^{x_2} \left(y^2 - \dot{y}^2\right) dx$
    where x is a parameter that defines the path: $y = y(x)$ and $\dot{y} = \frac{dy}{dx}$.
11. The perihelion (closest point from Sun) of Mercury is $r_p = 46 \times 10^6 \, \text{km}$ when it has a velocity of $v_p = 58.98 \, \text{km s}^{-1}$. What is the velocity of Mercury at its aphelion (farthest from the sun) which is at a distance of $r_a = 69.82 \times 10^6 \, \text{km}$?
12. A block of mass m is in vertical free fall. Write down its
    (a) Hamiltonian
    (b) Hamilton's equations of motion. [2+3]
13. The D'Alembert solution for waves on a string is given by: $y(x,t) = f(x-ct) + g(x+ct)$ where $f(x-ct)$ represents the forward moving mode and $g(x+ct)$ represents the negative moving mode.
    (a) Obtain the solution for the special case of a semi-infinite string that is fixed at $x=0$.
    (b) What is the physical interpretation of this solution? [3+2]