Date:

Reg. number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
B.Sc. MATHEMATICS - III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2021
(Examination conducted in JANUARY-MARCH 2022)
MT 318 – MATHEMATICS III

Time- 2 ½ hrs Max Marks: 70

This question paper contains **TWO** printed pages and **FOUR** parts

I. Answer any FIVE of the following 5x2=10

1) Define Order of an element.
2) Write any two generators of $(\mathbb{Z}_5, \oplus_5)$.
3) Find any two distinct cosets of $H = \{0,3,6\}$ in $(\mathbb{Z}_9, \oplus_9)$.
4) Find the kernel of the homomorphism $\phi: (\mathbb{R}^*, \times) \longrightarrow (\mathbb{R}^*, \times)$ defined by $\phi(x) = |x|, \forall\, x \in \mathbb{R}^*$, where $\mathbb{R}^*$ is the set of all non-zero real numbers.
5) Check if the function $f(x)$ is continuous at $x = 1$ or not.
$$f(x) = \begin{cases} x^2 + 2\,, & x > 1 \\ 2x + 1\,, & x = 1 \\ 3\,, & x < 1 \end{cases}$$
6) What is the upper bound and the lower bound of the set of natural numbers?
7) Find the critical point of $f(x, y) = 2x^2 - xy + y^2 + 7x$.
8) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$.

II. Answer any THREE of the following 3x6=18

9) Define cyclic group. Let G be a group, prove that for any $a \in G$, $< a > = < a^{-1} >$.
10) State and prove Lagrange's theorem for finite groups.
11) Define center of a group. Show that the center of a group G is normal in G.
12) Let $\phi: G \longrightarrow G'$ be a group homomorphism and H be a subgroup of G, then prove that If H is cyclic, then $\phi(H)$ is also cyclic.
13) State and prove fundamental theorem of homomorphism of groups.

III. Answer any FOUR of the following 4x6=24

14) Prove that if a function $f(x)$ is continuous on $[a, b]$ then the function attains its bounds at least once in $[a, b]$.
15) State and prove Cauchy's mean value theorem.
16) Expand the function $log_e(1 + e^x)$ using Maclaurin's expansion up to the terms containing x^4.
17) Find the maxima and minima of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

18) Find the volume of the largest rectangular parallelopiped than can be inscribed in the ellipsoid using the method of Lagrange multiplier.

IV. **Answer any THREE of the following** **3x6=18**

19) Solve the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = sin\,(4x)$.

20) Solve the system of differential equations:

$$\frac{dx}{dt} = 3x - y\ ;\quad \frac{dy}{dx} = x + y$$

21) Solve $x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$ when part of the complimentary function is e^x.

22) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + 9y = sec\,(3x)$.
