Register Number:

Date:

ST. JOSEPH's COLLEGE (Autonomous), BENGALURU - 27
M.Sc Mathematics-II Semester
Semester Examination: April 2022
(Examination conducted in July 2022)
MT-8321: Complex Analysis

Duration: $2\frac{1}{2}$ Hours **Max. Marks:** 70

1. The paper contains two pages.
2. Answer any **SEVEN** questions.
3. Each question carries 10 marks.

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1. State and prove the Rectangle Theorem II. [10]
2. (a) Find the value of the integral of $f(z) = \dfrac{1}{z^2+4}$ around the circle $|z-i| = 2$ oriented counterclock-wise.
   (b) Find the value of the integral of $f(z) = \dfrac{1}{2z-z^2}$ along the curve $|z| = 1$ oriented counterclock-wise.

   [5+5]
3. If f is analytic in $D(\alpha, r)$, then show that there exist constants C_k such that $f(z) = \sum_{k=0}^{\infty} C_k(z-\alpha)^k$ for all $z \in D(\alpha, r)$. [10]
4. State and prove Uniqueness Theorem. [10]
5. State and prove Open Mapping Theorem. [10]
6. Classify the singularities of following function and calculate the resdiue.
   (a) $f(z) = \dfrac{e^z}{1-z^2}$
   (b) $f(z) = (1-z^2)e^{1/z}$

   [5+5]

7. Let $f(z) = \dfrac{z^2}{z^2 - z - 2}$. Find the Laurent series of $f(z)$ in the domains $1 < |z| < 2$ and $0 < |z - 2| < 1$. [10]

8. State and prove Cauchy's Resdiue Theorem. [10]

9. Evaluate

   (a) $\displaystyle\int_0^\infty \frac{x^2 dx}{(x^2+4)^2(x^2+9)}$

   (b) $\displaystyle\int_{-\infty}^\infty \frac{x^2 dx}{(1+x^2)^2}$

   [7+3]

10. Suppose f is analytic at z_0 and $f'(z_0) \neq 0$. Then show that f is conformal and locally one-one at z_0. [10]