Register Number:
Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.SC MATHEMATICS - II SEMESTER
SEMESTER EXAMINATION: APRIL, 2022
(Examination conducted in July 2022)

MT 8518: TOPOLOGY

Duration: 2.5 Hours **Max. Marks:** 70

1. The paper contains two printed pages.
2. Answer any **SEVEN FULL** questions.
3. All true or false questions must be justified.

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1. a) Define co-finite topology on a topological space X. Prove that it is a topology. **[7 m]**
   b) True/False: The set of invertible $n \times n$ matrices is an open subset of the set of all $n \times n$ matrices. **[3 m]**
2. a) If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a topology of Y then show that the collection $\mathcal{D} = \{B \times C : B \in \mathcal{B} \text{ and } C \in \mathcal{C}\}$ is a basis for the product topology of $X \times Y$. **[4 m]**
   b) Let X be a Hausdorff space. Let $\Delta = \{(x, x) : x \in X\}$. Show that Δ is closed in $X \times X$. **[4 m]**
   c) True/False: Let X be a topological space and $A, B \subseteq X$, then $(A - B)^\circ = A^\circ - B^\circ$ **[2 m]**
3. a) State and prove the pasting lemma. **[7 m]**
   b) True/False: A bijective continuous map is a homeomorphism. **[3 m]**
4. a) Let $f : X \to Y$ be a function. Show that the following are equivalent:
   i. f is continuous.
   ii. For every subset A of X, one has $f(\overline{A}) \subseteq \overline{f(A)}$.
   iii. For every closed set B of Y, the set $f^{-1}(B)$ is closed in X. **[8 m]**
   b) True/False: A continous bijection is a homeomorphism. **[2 m]**
5. a) Let $\{A_\alpha\}$ be a collection of connected subsets of X such that $\bigcap_\alpha A_\alpha$ is non-empty. Show that $\bigcup_\alpha A_\alpha$ is connected. **[3 m]**
   b) Show that the continuous image of a connected set is connected. **[5 m]**
   c) True/False: Let X be connected and $Y \subsetneq X$ be a non-empty connected subspace of X. Then $X - Y$ is connected. **[2 m]**
6. a) Show that an open connected subset U of $\mathbb{R}^n$ is path connected. **[7 m]**
   b) True/False: The set of all $n \times n$ real diagonal matrices is connected. **[3 m]**

7. a) Show that a compact subset of a Hausdorff space is closed. **[6 m]**

   b) Show that any set with the co-finite topology is compact. **[4 m]**

8. a) State and prove the tube lemma. **[7 m]**

   b) True/False: The set of orthogonal matrices (matrices satisfying $AA^T = I = A^T A$) is a compact subset of $M_n(\mathbb{R})$, the set of $n \times n$ matrices with real entries. **[3 m]**

9. a) Suppose X has a countable basis then show that every open covering of X contains a countable subcollection covering X. **[4 m]**

   b) Prove that every compact Hausdorff space is normal. **[6 m]**

10. a) Show that the subspace of a Hausdorff space is Hausdorff. **[4 m]**

    b) Let X be a topological space in which one-point sets are closed. Show that X is regular if and only if given a point $x \in X$ and a neighbourhood U of x, there exists a neighbourhood V of x such that $\bar{V} \subseteq U$. **[6 m]**