Date:
Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.Sc. MATHEMATICS - II SEMESTER
SEMESTER EXAMINATION: APRIL 2022
(Examination conducted in July 2022)

MT 8521 – TOPOLOGY

Time- 2 ½ hrs Max Marks-70

This question paper contains **TWO** printed sides and **ONE** part.

Answer any 7 questions:

1. **A.** If A is a subset of a topological space X, prove that $x \in \bar{A}$ if and only if every open set U containing x intersects A.
   B. Let $X = \{1,2,3\}$ and $\mathfrak{B} = \{\{1\}, \{2,3\}, \{3\}\}$. Show that $\mathfrak{B}$ is a basis and find the topology generated by $\mathfrak{B}$. **[5m+5m]**

2. **A**. Define closure and interior of a set.
   If $X = \{a, b, c, d\}$ and $\tau = \{\phi, X, \{a\}, \{a, b\}, \{d\}, \{a, d\}, \{a, b, d\}\}$ then find
   i. The interior of A, if $A = \{a, b, c\}$
   ii. The closure of B, if $B = \{d\}$

   B. Let X be a Hausdorff space. Prove that a sequence of points in X converges to at most one point of X. **[5m+5m]**

3. **A**. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x + 1$ is a homeomorphism.
   B. If X and Y are topological spaces and if $f: X \to Y$ then prove the following are equivalent.
   a. f is continuous.
   b. For every subset A of X, $f(\bar{A}) \subset \overline{f(A)}$.
   c. For every closed set B of Y, the set $f^{-1}(B)$ is closed in X. **[3m+7m]**

4. **A**. Let $f: X \to Y$ and $g: Y \to Z$ be continuous functions. Show that $g \circ f: X \to Z$ is continuous.
   B. State and prove the pasting lemma. **[3m+7m]**

5. **A**. Define connected space. Give an example with justification.
   B. Prove that the union of a collection of subsets of X that have a point in common is connected. **[3m+7m]**

6. **A**. Show that $(0, 1)$ is not homeomorphic to (0,1].
   B. Show that a path connected set is connected.
   C. True/False. A totally disconnected space must have the discrete topology.
   [3m+5m+2m]

7. **A**. Prove that every compact subspace of a Hausdorff space is closed.
   B. Let $f: X \to Y$ be a bijective continuous function. If X is compact and Y is Hausdorff, then prove that f is homeomorphism. **[6m+4m]**

8. **A**. Show that $\{0\} \cup \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ is a compact subset of $\mathbb{R}$ with usual topology.
   B. Prove that the product of finitely many compact spaces is compact. **[3m+7m]**

9. **A**. Prove that a subspace of a first-countable space is first-countable.
   B. If X has a countable basis then prove that
   i. Every open covering of X contains a countable subcollection covering X.
   ii. There exists a countable subset of X that is dense in X. **[4m+6m]**

10. **A**. Let X be a topological space. Let one-point sets in X be closed. Prove that X is regular if and only if given a point x of X and a neighbourhood U of x, there is a neighbourhood V of x such that $\bar{V} \subset U$.
    B. Prove that every metrizable space is normal. **[5m+5m]**