Date:
Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
B.Sc. MATHEMATICS - VI SEMESTER
SEMESTER EXAMINATION: APRIL 2022
(Examination conducted in July 2022)

MT 6115 – Mathematics VII

Time- 2 ½ hrs Max Marks-70

This question paper contains one printed page and three parts.

I. ANSWER ANY FIVE OF THE FOLLOWING: (5x2=10)

1. Express (3,5,2) as a linear combination of the vectors (1,1,0), (2,3,0), (0,0,1) of $V_3(R)$.
2. Define subspace of a vector space.
3. Find the linear transformation $T: R^2 \to R^2$ such that $T(1,0) = (1,1) and\ T(0,1) = (-1,2)$
4. Find the scalar factors for cylindrical polar coordinates.
5. Solve $\frac{dx}{y^2z} = \frac{dy}{x^2z} = \frac{dz}{y^2x}$
6. Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$
7. Solve $(p + q)(z - xp - yq) = 1$
8. Solve $x^2p + y^2q = z^2$

II. ANSWER ANY THREE OF THE FOLLOWING: (3x6=18)

9. Find the dimension and basis of the subspace spanned by the vectors S = {(2,4,2), (1, -1,0), (1,2,1), (0,3,1)} in $V_3(R)$.
10. Find the matrix of the linear transformation $T: V_2(R) \to V_3(R)$ defined by $T(x,y) = (2y - x, y, 3y - 3x)$ relative to bases $B_1 = \{(1,1), (-1,1)\}$ and $B_2 = \{(1,1,1), (1,-1,1), (0,0,1)\}$
11. For the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ find the corresponding linear transformation $T: R^2 \to R^2$ with respect to the basis {(1,0), (1,1)}
12. State and prove Rank-Nullity theorem.

III. ANSWER ANY SEVEN OF THE FOLLOWING: (7x6=42)

13. Show that the cylindrical coordinate system is an orthogonal curvilinear system.
14. Derive the expression for the unit vectors $\widehat{e_\rho}$, $\widehat{e_\theta}$, $\widehat{e_\varphi}$ in the spherical coordinate system.
15. Verify the condition for integrability and solve $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0$
16. Form the partial differential equation for $z = yf(x) + xg(y)$, where $f\ and\ g$ are arbitrary functions.
17. Solve $zxp + yzq = xy$
18. Find the complete integral of $px + qy = pq$ by Charpit's method.
19. Solve $(D^2 - 3DD' + 2D'^2)z = e^{x+y}$
20. Solve $\left(D^2 - 2DD' + D'^2\right)z = xy$
21. Derive the Fourier series solution of the one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$